## An Online CPD Course brought to you by CEDengineering.ca

## Alternating Current (AC) Systems

Course No: E08-001
Credit: 8 PDH

Gilbert Gedeon, P.E.

Continuing Education and Development, Inc.
P: (877) 322-5800
info@cedengineering.ca

This course was adapted from the Department of Energy, Publication No. DOE-HDBK-1011/2-92, "Electrical Science", Modules 7 to 13, which is in the public domain.

## PART 1: BASIC AC THEORY - TABLE OF CONTENTS

REFERENCES ..... ii
AC GENERATION ..... 1
Development of a Sine-Wave Output ..... 1
Summary ..... 3
AC GENERATION ANALYSIS ..... 4
Effective Values ..... 4
Phase Angle ..... 7
Voltage Calculations ..... 8
Current Calculations ..... 9
Frequency Calculations ..... 9
Summary ..... 10

## REFERENCES

- Gussow, Milton, Schaum's Outline Series, Basic Electricity, McGraw-Hill.
- Academic Program for Nuclear Power Plant Personnel, Volume IV, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Sienko and Plane, Chemical Principles and Properties, $2^{\text {nd }}$ Edition, McGraw-Hill.
- Academic Program for Nuclear Power Plant Personnel, Volume II, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1981.
- Nasar and Unnewehr, Electromechanics and Electric Machines, John Wiley and Sons.
- Van Valkenburgh, Nooger, and Neville, Basic Electricity, Vol. 5, Hayden Book Company.
- Exide Industrial Marketing Division, The Storage Battery, Lead-Acid Type, The Electric Storage Battery Company.
- Lister, Eugene C., Electric Circuits and Machines, $5^{\text {th }}$ Edition, McGraw-Hill.
- Croft, Carr, Watt, and Summers, American Electricians Handbook, $10^{\text {th }}$ Edition, McGraw-Hill.
- Mason, C. Russel, The Art and Science of Protective Relaying, John Wiley and Sons.
- Mileaf, Harry, Electricity One - Seven, Revised $2^{\text {nd }}$ Edition, Hayden Book Company.
- Buban and Schmitt, Understanding Electricity and Electronics, $3{ }^{\text {rd }}$ Edition, McGrawHill.
- Kidwell, Walter, Electrical Instruments and Measurements, McGraw-Hill.


## AC GENERATION

An understanding of how an AC generator develops an AC output will help the student analyze the $A C$ power generation process.

EO 1.1 DESCRIBE the construction and operation of a simple AC generator.

EO 1.2 EXPLAIN the development of a sine-wave output in an AC generator.

The elementary AC generator (Figure 1) consists of a conductor, or loop of wire in a magnetic field that is produced by an electromagnet. The two ends of the loop are connected to slip rings, and they are in contact with two brushes. When the loop rotates it cuts magnetic lines of force, first in one direction and then the other.


Figure 1 Simple AC Generator

## Development of a Sine-Wave Output

At the instant the loop is in the vertical position (Figure 2, $0^{\circ}$ ), the coil sides are moving parallel to the field and do not cut magnetic lines of force. In this instant, there is no voltage induced in the loop. As the coil rotates in a counter-clockwise direction, the coil sides will cut the magnetic lines of force in opposite directions. The direction of the induced voltages depends on the direction of movement of the coil.

The induced voltages add in series, making slip ring X (Figure 1) positive (+) and slip ring Y (Figure 1) negative (-). The potential across resistor R will cause a current to flow from Y to X through the resistor. This current will increase until it reaches a maximum value when the coil is horizontal to the magnetic lines of force (Figure 2, $90^{\circ}$ ). The horizontal coil is moving perpendicular to the field and is cutting the greatest number of magnetic lines of force. As the coil continues to turn, the voltage and current induced decrease until they reach zero, where the coil is again in the vertical position (Figure 2, $180^{\circ}$ ). In the other half revolution, an equal voltage is produced except that the polarity is reversed (Figure 2, $270^{\circ}, 360^{\circ}$ ). The current flow through R is now from X to Y (Figure 1).


Figure 2 Developing a Sine-Wave Voltage

The periodic reversal of polarity results in the generation of a voltage, as shown in Figure 2. The rotation of the coil through $360^{\circ}$ results in an AC sine wave output.

## Summary

AC generation is summarized below.

## AC Generation Summary

- A simple generator consists of a conductor loop turning in a magnetic field, cutting across the magnetic lines of force.
- The sine wave output is the result of one side of the generator loop cutting lines of force. In the first half turn of rotation this produces a positive current and in the second half of rotation produces a negative current. This completes one cycle of AC generation.


## AC GENERATION ANALYSIS

Analysis of the AC power generation process and of the alternating current we use in almost every aspect of our lives is necessary to better understand how AC power is used in today's technology.

EO 1.3 DEFINE the following terms in relation to AC generation:
a. Radians/second
b. Hertz
c. Period

EO 1.4 DEFINE effective value of an AC current relative to DC current.

EO 1.5 Given a maximum value, CALCULATE the effective (RMS) and average values of AC voltage.

EO 1.6 Given a diagram of two sine waves, DESCRIBE the phase relationship between the two waves.

## Effective Values

The output voltage of an AC generator can be expressed in two ways. One is graphically by use of a sine wave (Figure 3). The second way is algebraically by the equation $\mathrm{e}=\mathrm{E}_{\text {max }} \sin \omega \mathrm{t}$, which will be covered later in the text.


Figure 3 Voltage Sine Wave

When a voltage is produced by an AC generator, the resulting current varies in step with the voltage. As the generator coil rotates $360^{\circ}$, the output voltage goes through one complete cycle. In one cycle, the voltage increases from zero to $\mathrm{E}_{\text {max }}$ in one direction, decreases to zero, increases to $\mathrm{E}_{\max }$ in the opposite direction (negative $\mathrm{E}_{\max }$ ), and then decreases to zero again. The value of $\mathrm{E}_{\text {max }}$ occurs at $90^{\circ}$ and is referred to as peak voltage. The time it takes for the generator to complete one cycle is called the period, and the number of cycles per second is called the frequency (measured in hertz).

One way to refer to AC voltage or current is by peak voltage $\left(E_{p}\right)$ or peak current $\left(I_{p}\right)$. This is the maximum voltage or current for an AC sine wave.

Another value, the peak-to-peak value ( $\mathrm{E}_{\mathrm{p}-\mathrm{p}}$ or $\mathrm{I}_{\mathrm{p}-\mathrm{p}}$ ), is the magnitude of voltage, or current range, spanned by the sine wave. However, the value most commonly used for AC is effective value. Effective value of AC is the amount of AC that produces the same heating effect as an equal amount of DC. In simpler terms, one ampere effective value of AC will produce the same amount of heat in a conductor, in a given time, as one ampere of DC. The heating effect of a given AC current is proportional to the square of the current. Effective value of AC can be calculated by squaring all the amplitudes of the sine wave over one period, taking the average of these values, and then taking the square root. The effective value, being the root of the mean (average) square of the currents, is known as the root-mean-square, or RMS value. In order to understand the meaning of effective current applied to a sine wave, refer to Figure 4.

The values of I are plotted on the upper curve, and the corresponding values of $\mathrm{I}^{2}$ are plotted on the lower curve. The $\mathrm{I}^{2}$ curve has twice the frequency of I and varies above and below a new axis. The new axis is the average of the $I^{2}$ values, and the square root of that value is the RMS, or effective value, of current. The average value is $1 / 2 \mathrm{I}_{\text {max }}^{2}$. The RMS value is then $\frac{\sqrt{ } 2 \mathrm{I}_{\max }^{2}}{2}$ OR $\frac{\sqrt{ } 2}{2} \mathrm{I}_{\max }$, which is equal to $0.707 \mathrm{I}_{\max }$.

There are six basic equations that are used to convert a value of AC voltage or current to another value, as listed below.

Average value $=$ peak value $\times 0.637$
Effective value (RMS) = peak value x 0.707
Peak value $=$ average value x 1.57
Effective value $($ RMS $)=$ average value $\times 1.11$
Peak value $=$ effective value (RMS) x 1.414
Average value $=$ effective $($ RMS $) \times 0.9$
The values of current (I) and voltage (E) that are normally encountered are assumed to be RMS values; therefore, no subscript is used.


Figure 4 Effective Value of Current
Another useful value is the average value of the amplitude during the positive half of the cycle. Equation (7-7) is the mathematical relationship between $\mathrm{I}_{\mathrm{av}}, \mathrm{I}_{\max }$, and I.

$$
\begin{equation*}
\mathrm{I}_{\mathrm{av}}=0.637 \mathrm{I}_{\max }=0.90 \mathrm{I} \tag{7-7}
\end{equation*}
$$

Equation (7-8) is the mathematical relationship between $\mathrm{E}_{\mathrm{av}}, \mathrm{E}_{\max }$, and E .

$$
\begin{equation*}
\mathrm{E}_{\mathrm{av}}=0.637 \mathrm{E}_{\max }=0.90 \mathrm{E} \tag{7-8}
\end{equation*}
$$

Example 1: The peak value of voltage in an AC circuit is 200 V . What is the RMS value of the voltage?

$$
\begin{aligned}
& \mathrm{E}=0.707 \mathrm{E}_{\max } \\
& \mathrm{E}=0.707(200 \mathrm{~V}) \\
& \mathrm{E}=141.4 \mathrm{~V}
\end{aligned}
$$

Example 2: The peak current in an AC circuit is 10 amps . What is the average value of current in the circuit?

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{av}}=0.637 \mathrm{I}_{\max } \\
& \mathrm{I}_{\mathrm{av}}=0.637(10 \mathrm{amps}) \\
& \mathrm{I}_{\mathrm{av}}=6.37 \mathrm{amps}
\end{aligned}
$$

## Phase Angle

Phase angle is the fraction of a cycle, in degrees, that has gone by since a voltage or current has passed through a given value. The given value is normally zero. Referring back to Figure 3, take point 1 as the starting point or zero phase. The phase at Point 2 is $30^{\circ}$, Point 3 is $60^{\circ}$, Point 4 is $90^{\circ}$, and so on, until Point 13 where the phase is $360^{\circ}$, or zero. A term more commonly used is phase difference. The phase difference can be used to describe two different voltages that have the same frequency, which pass through zero values in the same direction at different times. In Figure 5, the angles along the axis indicate the phases of voltages $e_{1}$ and $e_{2}$ at any point in time. At $120^{\circ}$, $\mathrm{e}_{1}$ passes through the zero value, which is $60^{\circ}$ ahead of $\mathrm{e}_{2}$ ( $\mathrm{e}_{2}$ equals zero at $180^{\circ}$ ). The voltage $e_{1}$ is said to lead $e_{2}$ by 60 electrical degrees, or it can be said that $e_{2}$ lags $e_{1}$ by 60 electrical degrees.


Figure 5 Phase Relationship

Phase difference is also used to compare two different currents or a current and a voltage. If the phase difference between two currents, two voltages, or a voltage and a current is zero degrees, they are said to be "in-phase." If the phase difference is an amount other than zero, they are said to be "out-of-phase."

## Voltage Calculations

Equation (7-9) is a mathematical representation of the voltage associated with any particular orientation of a coil (inductor).

$$
\begin{equation*}
\mathrm{e}=\mathrm{E}_{\max } \sin \theta \tag{7-9}
\end{equation*}
$$

where
e = induced EMF
$\mathrm{E}_{\text {max }}=$ maximum induced EMF
$\theta \quad=$ angle from reference (degrees or radians)
Example 1: What is the induced EMF in a coil producing a maximum EMF of 120 V when the angle from reference is $45^{\circ}$ ?

$$
\begin{aligned}
& \mathrm{e}=\mathrm{E}_{\max } \sin \theta \\
& \mathrm{e}=120 \mathrm{~V}\left(\sin 45^{\circ}\right) \\
& \mathrm{e}=84.84 \mathrm{~V}
\end{aligned}
$$

The maximum induced voltage can also be called peak voltage $\mathrm{E}_{\mathrm{p}}$. If $(\mathrm{t})$ is the time in which the coil turns through the angle $(\theta)$, then the angular velocity $(\omega)$ of the coil is equal to $\theta / \mathrm{t}$ and is expressed in units of radians $/ \mathrm{sec}$. Equation (7-10) is the mathematical representation of the angular velocity.

$$
\begin{equation*}
\theta=\omega t \tag{7-10}
\end{equation*}
$$

where

$$
\begin{aligned}
& \omega=\text { angular velocity (radians } / \mathrm{sec} \text { ) } \\
& \mathrm{t}=\text { time to turn through the angle from reference (sec) } \\
& \theta=\text { angle from reference (radians) }
\end{aligned}
$$

Using substitution laws, a relationship between the voltage induced, the maximum induced voltage, and the angular velocity can be expressed. Equation (7-11) is the mathematical representation of the relationship between the voltage induced, the maximum voltage, and the angular velocity, and is equal to the output of an AC Generator.

$$
\begin{equation*}
\mathrm{e}=\mathrm{E}_{\max } \sin (\omega \mathrm{t}) \tag{7-11}
\end{equation*}
$$

where
e $\quad=\quad$ induced EMF (volts)
$\mathrm{E}_{\text {max }}=$ maximum induced EMF (volts)
$\omega \quad=\quad$ angular velocity (radians/sec)
$\mathrm{t} \quad=\quad$ time to turn through the angle from reference (sec)

## Current Calculations

Maximum induced current is calculated in a similar fashion. Equation (7-12) is a mathematical representation of the relationship between the maximum induced current and the angular velocity.

$$
\begin{equation*}
i=I_{\max } \sin (\omega t) \tag{7-12}
\end{equation*}
$$

where

```
i \(\quad=\) induced current (amps)
\(\mathrm{I}_{\text {max }}=\) maximum induced current (amps)
\(\omega \quad=\) angular velocity (radians/sec)
\(\mathrm{t} \quad=\) time to turn through the angle from reference (sec)
```


## Frequency Calculations

The frequency of an alternating voltage or current can be related directly to the angular velocity of a rotating coil. The units of angular velocity are radians per second, and $2 \pi$ radians is a full revolution. A radian is an angle that subtends an arc equal to the radius of a circle. One radian equals 57.3 degrees. One cycle of the sine wave is generated when the coil rotates $2 \pi$ radians. Equation (7-13) is the mathematical relationship between frequency (f) and the angular velocity $(\omega)$ in an AC circuit.

$$
\begin{equation*}
\omega=2 \pi f \tag{7-13}
\end{equation*}
$$

where

$$
\begin{aligned}
& \omega=\text { angular velocity (radians/sec) } \\
& \mathrm{f}=\text { frequency }(\mathrm{HZ})
\end{aligned}
$$

Example 1: The frequency of a 120 V AC circuit is 60 Hz . Find the following:

1. Angular velocity
2. Angle from reference at 1 msec
3. Induced EMF at that point

Solution:

$$
\begin{aligned}
& \text { 1. } \omega=2 \pi \mathrm{f} \\
& =2(3.14)(60 \mathrm{~Hz}) \\
& =376.8 \text { radians } / \mathrm{sec} \\
& \text { 2. } \theta=\omega t \\
& =(376.8 \mathrm{radian} / \mathrm{sec})(.001 \mathrm{sec}) \\
& =0.3768 \text { radians } \\
& \text { 3. } \mathrm{e}=\mathrm{E}_{\max } \sin \theta \\
& =(120 \mathrm{~V})(\sin 0.3768 \text { radians }) \\
& =(120 \mathrm{~V})(0.3679) \\
& =44.15 \mathrm{~V}
\end{aligned}
$$

## Summary

AC generation analysis is summarized below.

## Voltage, Current, and Frequency Summary

- The following terms relate to the AC cycle: radians/second, the velocity the loop turns; hertz, the number of cycles in one second; period, the time to complete one cycle.
- Effective value of AC equals effective value of DC.
- Root mean square (RMS) values equate AC to DC equivalents:
- $\mathrm{I}=0.707 \mathrm{I}_{\max }=$ Effective Current
- $\mathrm{E}=0.707 \mathrm{E}_{\max }=$ Effective Voltage
- $\mathrm{I}_{\mathrm{av}}=0.636 \mathrm{I}_{\text {max }}=0.9 \mathrm{I}=$ Average Current
- $\mathrm{E}_{\mathrm{av}}=0.636 \mathrm{E}_{\max }=0.9 \mathrm{E}=$ Average Voltage
- Phase angle is used to compare two wave forms. It references the start, or zero point, of each wave. It compares differences by degrees of rotation. Wave forms with the same start point are "in-phase" while wave forms "out-of-phase" either lead or lag.
REFERENCES ..... ii
INDUCTANCE ..... 1
Inductive Reactance ..... 1
Voltage and Current Phase Relationships in an Inductive Circuit ..... 2
Summary ..... 4
CAPACITANCE ..... 5
Capacitors ..... 5
Capacitive Reactance ..... 6
Summary ..... 8
IMPEDANCE ..... 9
Impedance ..... 9
Impedance in R-L Circuits ..... 11
Impedance in R-C Circuits ..... 12
Impedance in R-C-L Circuits ..... 13
Summary ..... 18
RESONANCE ..... 19
Resonant Frequency ..... 19
Series Resonance ..... 19
Parallel Resonance ..... 20
Summary ..... 20


## REFERENCES

- Gussow, Milton, Schaum's Outline Series, Basic Electricity, McGraw-Hill.
- Academic Program for Nuclear Power Plant Personnel, Volume IV, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Sienko and Plane, Chemical Principles and Properties, $2^{\text {nd }}$ Edition, McGraw-Hill.
- Academic Program for Nuclear Power Plant Personnel, Volume II, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Nasar and Unnewehr, Electromechanics and Electric Machines, John Wiley and Sons.
- Van Valkenburgh, Nooger, and Neville, Basic Electricity, Vol. 5, Hayden Book Company.
- Exide Industrial Marketing Division, The Storage Battery, Lead-Acid Type, The Electric Storage Battery Company.
- Lister, Eugene C., Electric Circuits and Machines, $5^{\text {th }}$ Edition, McGraw-Hill.
- Croft, Carr, Watt, and Summers, American Electricians Handbook, $10^{\text {th }}$ Edition, McGraw-Hill.
- Mason, C. Russel, The Art and Science of Protective Relaying, John Wiley and Sons.
- Mileaf, Harry, Electricity One - Seven, Revised $2^{\text {nd }}$ Edition, Hayden Book Company.
- Buban and Schmitt, Understanding Electricity and Electronics, $3{ }^{\text {rd }}$ Edition, McGrawHill.
- Kidwell, Walter, Electrical Instruments and Measurements, McGraw-Hill.


## INDUCTANCE

Any device relying on magnetism or magnetic fields to operate is a form of inductor. Motors, generators, transformers, and coils are inductors. The use of an inductor in a circuit can cause current and voltage to become out-of-phase and inefficient unless corrected.

EO 1.1 DESCRIBE inductive reactance ( $\mathrm{X}_{\mathrm{L}}$ ).
EO 1.2 Given the operation frequency (f) and the value of inductance ( L ), CALCULATE the inductive reactance $\left(X_{L}\right)$ of a simple circuit.

EO 1.3 DESCRIBE the effect of the phase relationship between current and voltage in an inductive circuit.

EO 1.4 DRAW a simple phasor diagram representing AC current ( $I$ ) and voltage ( E ) in an inductive circuit.

## Inductive Reactance

In an inductive AC circuit, the current is continually changing and is continuously inducing an EMF. Because this EMF opposes the continuous change in the flowing current, its effect is measured in ohms. This opposition of the inductance to the flow of an alternating current is called inductive reactance $\left(\mathrm{X}_{\mathrm{L}}\right)$. Equation (8-1) is the mathematical representation of the current flowing in a circuit that contains only inductive reactance.

$$
\begin{equation*}
I=\frac{E}{X_{L}} \tag{8-1}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{I}=\text { effective current }(\mathrm{A}) \\
& \mathrm{X}_{\mathrm{L}}=\text { inductive reactance }(\Omega) \\
& \mathrm{E}=\text { effective voltage across the reactance }(\mathrm{V})
\end{aligned}
$$

The value of $\mathrm{X}_{\mathrm{L}}$ in any circuit is dependent on the inductance of the circuit and on the rate at which the current is changing through the circuit. This rate of change depends on the frequency of the applied voltage. Equation (8-2) is the mathematical representation for $\mathrm{X}_{\mathrm{L}}$.

$$
\begin{equation*}
X_{L}=2 \pi \mathrm{fL} \tag{8-2}
\end{equation*}
$$

where

$$
\begin{aligned}
& \pi=\sim 3.14 \\
& \mathrm{f}=\text { frequency (Hertz) } \\
& \mathrm{L}=\text { inductance (Henries) }
\end{aligned}
$$

The magnitude of an induced EMF in a circuit depends on how fast the flux that links the circuit is changing. In the case of self-induced EMF (such as in a coil), a counter EMF is induced in the coil due to a change in current and flux in the coil. This CEMF opposes any change in current, and its value at any time will depend on the rate at which the current and flux are changing at that time. In a purely inductive circuit, the resistance is negligible in comparison to the inductive reactance. The voltage applied to the circuit must always be equal and opposite to the EMF of self-induction.

## Voltage and Current Phase Relationships in an Inductive Circuit

As previously stated, any change in current in a coil (either a rise or a fall) causes a corresponding change of the magnetic flux around the coil. Because the current changes at its maximum rate when it is going through its zero value at $90^{\circ}$ (point b on Figure 1) and $270^{\circ}$ (point d), the flux change is also the greatest at those times. Consequently, the self-induced EMF in the coil is at its maximum (or minimum) value at these points, as shown in Figure 1. Because the current is not changing at the point when it is going through its peak value at $0^{\circ}$ (point a), $180^{\circ}$ (point c), and $360^{\circ}$ (point e), the flux change is zero at those times. Therefore, the selfinduced EMF in the coil is at its zero value at these points.


Figure 1 Current, Self-Induced EMF, and Applied Voltage in an Inductive Circuit

According to Lenz's Law (refer to Module 1, Basic Electrical Theory), the induced voltage always opposes the change in current. Referring to Figure 1, with the current at its maximum negative value (point a), the induced EMF is at a zero value and falling. Thus, when the current rises in a positive direction (point a to point c), the induced EMF is of opposite polarity to the applied voltage and opposes the rise in current. Notice that as the current passes through its zero value (point $b$ ) the induced voltage reaches its maximum negative value. With the current now at its maximum positive value (point c), the induced EMF is at a zero value and rising. As the current is falling toward its zero value at $180^{\circ}$ (point c to point d), the induced EMF is of the same polarity as the current and tends to keep the current from falling. When the current reaches a zero value, the induced EMF is at its maximum positive value. Later, when the current is increasing from zero to its maximum negative value at $360^{\circ}$ (point d to point e ), the induced voltage is of the opposite polarity as the current and tends to keep the current from increasing in the negative direction. Thus, the induced EMF can be seen to lag the current by $90^{\circ}$.

The value of the self-induced EMF varies as a sine wave and lags the current by $90^{\circ}$, as shown in Figure 1. The applied voltage must be equal and opposite to the self-induced EMF at all times; therefore, the current lags the applied voltage by $90^{\circ}$ in a purely inductive circuit.

If the applied voltage ( E ) is represented by a vector rotating in a counterclockwise direction (Figure 1b), then the current can be expressed as a vector that is lagging the applied voltage by $90^{\circ}$. Diagrams of this type are referred to as phasor diagrams.

Example: A 0.4 H coil with negligible resistance is connected to a $115 \mathrm{~V}, 60 \mathrm{~Hz}$ power source (see Figure 2). Find the inductive reactance of the coil and the current through the circuit. Draw a phasor diagram showing the phase relationship between current and applied voltage.


Figure 2 Coil Circuit and Phasor Diagram

Solution:

1. Inductive reactance of the coil

$$
\begin{aligned}
\mathrm{X}_{\mathrm{L}} & =2 \pi \mathrm{fL} \\
& =(2)(3.14)(60)(0.4) \\
\mathrm{X}_{\mathrm{L}} & =150.7 \Omega
\end{aligned}
$$

2. Current through the circuit

$$
\begin{aligned}
I & =\frac{E}{X_{L}} \\
& =\frac{115}{150.7}
\end{aligned}
$$

$\mathrm{I}=0.76 \mathrm{amps}$
3. Draw a phasor diagram showing the phase relationship between current and applied voltage.

Phasor diagram showing the current lagging voltage by $90^{\circ}$ is drawn in Figure 2 b .

## Summary

Inductive reactance is summarized below.

## Inductive Reactance Summary

- Opposition to the flow of alternating current caused by inductance is called Inductive Reactance ( $\mathrm{X}_{\mathrm{L}}$ ).
- The formula for calculating $X_{L}$ is:

$$
\mathrm{X}_{\mathrm{L}}=2 \pi \mathrm{fL}
$$

- Current (I) lags applied voltage (E) in a purely inductive circuit by $90^{\circ}$ phase angle.
- The phasor diagram shows the applied voltage (E) vector leading (above) the current (I) vector by the amount of the phase angle differential due to the relationship between voltage and current in an inductive circuit.


## CAPACITANCE

There are many natural causes of capacitance in AC power circuits, such as transmission lines, fluorescent lighting, and computer monitors. Normally, these are counteracted by the inductors previously discussed. However, where capacitors greatly outnumber inductive devices, we must calculate the amount of capacitance to add or subtract from an AC circuit by artificial means.

EO 1.5 DEFINE capacitive reactance ( $\mathbf{X}_{C}$ ).
EO 1.6 Given the operating frequency (f) and the value of capacitance (C), CALCULATE the capacitive reactance $\left(X_{C}\right)$ of a simple AC circuit.

EO 1.7 DESCRIBE the effect on phase relationship between current (I) and voltage ( $E$ ) in a capacitive circuit.

EO 1.8 DRAW a simple phasor diagram representing AC current (I) and voltage ( $E$ ) in a capacitive circuit.

## Capacitors

The variation of an alternating voltage applied to a capacitor, the charge on the capacitor, and the current flowing through the capacitor are represented by Figure 3.

The current flow in a circuit containing capacitance depends on the rate at which the voltage changes. The current flow in Figure 3 is greatest at points a, c, and e. At these points, the voltage is changing at its maximum rate (i.e., passing through zero). Between points a and $b$, the voltage and charge are increasing, and the current flow is into the capacitor, but decreasing in value. At point $b$, the capacitor is fully charged, and the current is zero. From points b to c , the voltage and charge are decreasing as the capacitor discharges, and its current flows in a direction opposite to the voltage. From points c to d, the capacitor begins to charge in the opposite direction, and the voltage and current are again in the same direction.


Figure 3 Voltage, Charge, and Current in a Capacitor

At point $d$, the capacitor is fully charged, and the current flow is again zero. From points $d$ to e, the capacitor discharges, and the flow of current is opposite to the voltage. Figure 3 shows the current leading the applied voltage by $90^{\circ}$. In any purely capacitive circuit, current leads applied voltage by $90^{\circ}$.

## Capacitive Reactance

Capacitive reactance is the opposition by a capacitor or a capacitive circuit to the flow of current. The current flowing in a capacitive circuit is directly proportional to the capacitance and to the rate at which the applied voltage is changing. The rate at which the applied voltage is changing is determined by the frequency of the supply; therefore, if the frequency of the capacitance of a given circuit is increased, the current flow will increase. It can also be said that if the frequency or capacitance is increased, the opposition to current flow decreases; therefore, capacitive reactance, which is the opposition to current flow, is inversely proportional to frequency and capacitance. Capacitive reactance $X_{C}$, is measured in ohms, as is inductive reactance. Equation (8-3) is a mathematical representation for capacitive reactance.

$$
\begin{equation*}
X_{C}=\frac{1}{2 \pi f C} \tag{8-3}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{f}=\text { frequency }(\mathrm{Hz}) \\
& \pi=\sim 3.14 \\
& \mathrm{C}=\text { capacitance (farads) }
\end{aligned}
$$

Equation (8-4) is the mathematical representation of capacitive reactance when capacitance is expressed in microfarads ( $\mu \mathrm{F}$ ).

$$
\begin{equation*}
X_{C}=\frac{1,000,000}{2 \pi f C} \tag{8-4}
\end{equation*}
$$

Equation (8-5) is the mathematical representation for the current that flows in a circuit with only capacitive reactance.

$$
\begin{equation*}
I=\frac{E}{X_{C}} \tag{8-5}
\end{equation*}
$$

where
I = effective current (A)
$\mathrm{E}=$ effective voltage across the capacitive reactance (V)
$\mathrm{X}_{\mathrm{C}}=$ capacitive reactance $(\Omega)$
Example: A $10 \mu \mathrm{~F}$ capacitor is connected to a $120 \mathrm{~V}, 60 \mathrm{~Hz}$ power source (see Figure 4). Find the capacitive reactance and the current flowing in the circuit. Draw the phasor diagram.


Figure 4 Circuit and Phasor Diagram

## Solution:

1. Capacitive reactance
$X_{C}=\frac{1,000,000}{2 \pi f C}$
$=\frac{1,000,000}{(2)(3.14)(60)(10)}$
$=\frac{1,000,000}{3768}$
$X_{C}=265.4 \Omega$
2. Current flowing in the circuit

$$
\begin{aligned}
I & =\frac{E}{X_{C}} \\
& =\frac{120}{265.4} \\
I & =0.452 \mathrm{amps}
\end{aligned}
$$

3. Phasor diagram showing current leading voltage by $90^{\circ}$ is drawn in Figure 4 b .

## Summary

Capacitive reactance is summarized below.

## Capacitive Reactance Summary

- Opposition to the flow of alternating current caused by capacitance is called capacitive reactance $\left(\mathrm{X}_{\mathrm{C}}\right)$.
- The formula for calculating $X_{C}$ is:

$$
X_{C}=\frac{1}{2 \pi f C}
$$

- Current (I) leads applied voltage by $90^{\circ}$ in a purely capacitive circuit.
- The phasor diagram shows the applied voltage (E) vector leading (below) the current (I) vector by the amount of the phase angle differential due to the relationship between voltage and current in a capacitive circuit.


## IMPEDANCE

Whenever inductive and capacitive components are used in an AC circuit, the calculation of their effects on the flow of current is important.

EO 1.9 DEFINE impedance (Z).
EO 1.10 Given the values for resistance ( $\mathbf{R}$ ) and inductance ( $L$ ) and a simple R-L series AC circuit, CALCULATE the impedance ( $Z$ ) for that circuit.

EO 1.11 Given the values for resistance ( $\mathbf{R}$ ) and capacitance ( $C$ ) and a simple R-C series AC circuit, CALCULATE the impedance ( $Z$ ) for that circuit.

EO 1.12 Given a simple R-C-L series AC circuit and the values for resistance ( $R$ ), inductive reactance $\left(X_{L}\right)$, and capacitive reactance ( $X_{C}$ ), CALCULATE the impedance $(Z)$ for that circuit.

EO 1.13 STATE the formula for calculating total current $\left(I_{T}\right)$ in a simple parallel R-C-L AC circuit.

EO 1.14 Given a simple R-C-L parallel AC circuit and the values for voltage $\left(V_{T}\right)$, resistance ( $R$ ), inductive reactance $\left(X_{L}\right)$, and capacitive reactance $\left(X_{C}\right)$, CALCULATE the impedance ( $Z$ ) for that circuit.

## Impedance

No circuit is without some resistance, whether desired or not. Resistive and reactive components in an AC circuit oppose current flow. The total opposition to current flow in a circuit depends on its resistance, its reactance, and the phase relationships between them. Impedance is defined as the total opposition to current flow in a circuit. Equation (8-6) is the mathematical representation for the magnitude of impedance in an AC circuit.

$$
\begin{equation*}
\mathrm{Z}=\sqrt{\mathrm{R}^{2}+\mathrm{X}^{2}} \tag{8-6}
\end{equation*}
$$

where
$\mathrm{Z}=$ impedance $(\Omega)$
$\mathrm{R}=$ resistance $(\Omega)$
$\mathrm{X}=$ net reactance $(\Omega)$
The relationship between resistance, reactance, and impedance is shown in Figure 5.


Figure 5 Relationship Between Resistance, Reactance, and Impedance

The current through a certain resistance is always in phase with the applied voltage. Resistance is shown on the zero axis. The current through an inductor lags applied voltage by $90^{\circ}$; inductive reactance is shown along the $90^{\circ}$ axis. Current through a capacitor leads applied voltage by $90^{\circ}$; capacitive reactance is shown along the $-90^{\circ}$ axis. Net reactance in an AC circuit is the difference between inductive and capacitive reactance. Equation (8-7) is the mathematical representation for the calculation of net reactance when $X_{L}$ is greater than $X_{C}$.
$X=X_{L}-X_{C}$
where
$\mathrm{X}=$ net reactance $(\Omega)$
$\mathrm{X}_{\mathrm{L}}=$ inductive reactance $(\Omega)$
$\mathrm{X}_{\mathrm{C}}=$ capacitive reactance $(\Omega)$

Equation (8-8) is the mathematical representation for the calculation of net reactance when $X_{C}$ is greater than $X_{L}$.

$$
\begin{equation*}
X=X_{C}-X_{L} \tag{8-8}
\end{equation*}
$$

Impedance is the vector sum of the resistance and net reactance ( X ) in a circuit, as shown in Figure 5. The angle $\theta$ is the phase angle and gives the phase relationship between the applied voltage and the current. Impedance in an AC circuit corresponds to the resistance of a DC circuit. The voltage drop across an AC circuit element equals the current times the impedance. Equation (8-9) is the mathematical representation of the voltage drop across an AC circuit.

$$
\begin{equation*}
\mathrm{V}=\mathrm{IZ} \tag{8-9}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{V}=\text { voltage drop }(\mathrm{V}) \\
& \mathrm{I}=\text { current }(\mathrm{A}) \\
& \mathrm{Z}=\text { impedance }(\Omega)
\end{aligned}
$$

The phase angle $\theta$ gives the phase relationship between current and the voltage.

## Impedance in R-L Circuits

Impedance is the resultant of phasor addition of R and $\mathrm{X}_{\mathrm{L}}$. The symbol for impedance is Z . Impedance is the total opposition to the flow of current and is expressed in ohms. Equation (8-10) is the mathematical representation of the impedance in an RL circuit.

$$
\begin{equation*}
\mathrm{Z}=\sqrt{\mathrm{R}^{2}+\mathrm{X}_{\mathrm{L}}^{2}} \tag{8-10}
\end{equation*}
$$

Example: If a $100 \Omega$ resistor and a $60 \Omega \mathrm{X}_{\mathrm{L}}$ are in series with a 115 V applied voltage (Figure 6), what is the circuit impedance?


Figure 6 Simple R-L Circuit

Solution:

$$
\begin{aligned}
\mathrm{Z} & =\sqrt{\mathrm{R}^{2}+\mathrm{X}_{\mathrm{L}}^{2}} \\
& =\sqrt{100^{2}+60^{2}} \\
& =\sqrt{10,000+3600} \\
& =\sqrt{13,600} \\
\mathrm{Z} & =116.6 \Omega
\end{aligned}
$$

## Impedance in R-C Circuits

In a capacitive circuit, as in an inductive circuit, impedance is the resultant of phasor addition of $R$ and $X_{C}$. Equation (8-11) is the mathematical representation for impedance in an R-C circuit.

$$
\begin{equation*}
\mathrm{Z}=\sqrt{\mathrm{R}^{2}+\mathrm{X}_{\mathrm{C}}^{2}} \tag{8-11}
\end{equation*}
$$

Example: $\quad$ A $50 \Omega X_{C}$ and a $60 \Omega$ resistance are in series across a 110 V source (Figure 7 ). Calculate the impedance.


Figure 7 Simple R-C Circuit

Solution:

$$
\begin{aligned}
Z & =\sqrt{R^{2}+X_{C}^{2}} \\
& =\sqrt{60^{2}+50^{2}} \\
& =\sqrt{3600+2500} \\
& =\sqrt{6100} \\
Z & =78.1 \Omega
\end{aligned}
$$

## Impedance in R-C-L Circuits

Impedance in an R-C-L series circuit is equal to the phasor sum of resistance, inductive reactance, and capacitive reactance (Figure 8).


Figure 8 Series R-C-L Impedance-Phasor

Equations (8-12) and (8-13) are the mathematical representations of impedance in an R-C-L circuit. Because the difference between $\mathrm{X}_{\mathrm{L}}$ and $\mathrm{X}_{\mathrm{C}}$ is squared, the order in which the quantities are subtracted does not affect the answer.

$$
\begin{align*}
& \mathrm{Z}=\sqrt{\mathrm{R}^{2}+\left(\mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{C}}\right)^{2}}  \tag{8-12}\\
& \mathrm{Z}=\sqrt{\mathrm{R}^{2}+\left(\mathrm{X}_{\mathrm{C}}-\mathrm{X}_{\mathrm{L}}\right)^{2}} \tag{8-13}
\end{align*}
$$

Example: Find the impedance of a series R-C-L circuit, when $\mathrm{R}=6 \Omega, \mathrm{X}_{\mathrm{L}}=20 \Omega$, and $\mathrm{X}_{\mathrm{C}}$ $=10 \Omega$ (Figure 9).


Figure 9 Simple R-C-L Circuit

## Solution:

$$
\begin{aligned}
Z & =\sqrt{R^{2}+\left(X_{L}-X_{C}\right)^{2}} \\
& =\sqrt{6^{2}+(20-10)^{2}} \\
& =\sqrt{6^{2}+10^{2}} \\
& =\sqrt{36+100} \\
& =\sqrt{136} \\
Z & =11.66 \Omega
\end{aligned}
$$

Impedance in a parallel R-C-L circuit equals the voltage divided by the total current. Equation (8-14) is the mathematical representation of the impedance in a parallel R-C-L circuit.

$$
\begin{equation*}
\mathrm{Z}_{\mathrm{T}}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{I}_{\mathrm{T}}} \tag{8-14}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{Z}_{\mathrm{T}}=\text { total impedance }(\Omega) \\
& \mathrm{V}_{\mathrm{T}}=\text { total voltage }(\mathrm{V}) \\
& \mathrm{I}_{\mathrm{T}}
\end{aligned}=\text { total current }(\mathrm{A})
$$

Total current in a parallel R-C-L circuit is equal to the square root of the sum of the squares of the current flows through the resistance, inductive reactance, and capacitive reactance branches of the circuit. Equations (8-15) and (8-16) are the mathematical representations of total current in a parallel R-C-L circuit. Because the difference between $\mathrm{I}_{\mathrm{L}}$ and $\mathrm{I}_{\mathrm{C}}$ is squared, the order in which the quantities are subtracted does not affect the answer.

$$
\begin{align*}
& I_{T}=\sqrt{I_{R}^{2}+\left(I_{C}-I_{L}\right)^{2}}  \tag{8-15}\\
& I_{T}=\sqrt{I_{R}^{2}+\left(I_{L}-I_{C}\right)^{2}} \tag{8-16}
\end{align*}
$$

where

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{T}}=\text { total current }(\mathrm{A}) \\
& \mathrm{I}_{\mathrm{R}}=\text { current through resistance leg of circuit (A) } \\
& \mathrm{I}_{\mathrm{C}}=\text { current through capacitive reactance leg of circuit (A) } \\
& \mathrm{I}_{\mathrm{L}}=\text { current through inductive reactance leg of circuit (A) }
\end{aligned}
$$

Example: $\quad$ A $200 \Omega$ resistor, a $100 \Omega X_{\mathrm{L}}$, and an $80 \Omega \mathrm{X}_{\mathrm{C}}$ are placed in parallel across a 120V AC source (Figure 10). Find: (1) the branch currents, (2) the total current, and (3) the impedance.


Figure 10 Simple Parallel R-C-L Circuit

## Solution:

1. Branch currents

$$
\begin{aligned}
\mathrm{I}_{\mathrm{R}} & =\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{R}} & \mathrm{I}_{\mathrm{L}} & =\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{X}_{\mathrm{L}}} & \mathrm{I}_{\mathrm{C}} & =\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{X}_{\mathrm{C}}} \\
& =\frac{120}{200} & & =\frac{120}{100} & & =\frac{120}{80} \\
\mathrm{I}_{\mathrm{R}} & =0.6 \mathrm{~A} & \mathrm{I}_{\mathrm{L}} & =1.2 \mathrm{~A} & \mathrm{I}_{\mathrm{C}} & =1.5 \mathrm{~A}
\end{aligned}
$$

2. Total current

$$
\begin{aligned}
\mathrm{I}_{\mathrm{T}} & =\sqrt{\mathrm{I}_{\mathrm{R}}^{2}+\left(\mathrm{I}_{\mathrm{C}}-\mathrm{I}_{\mathrm{L}}\right)^{2}} \\
& =\sqrt{(0.6)^{2}+(1.5-1.2)^{2}} \\
& =\sqrt{(0.6)^{2}+(0.3)^{2}} \\
& =\sqrt{0.36+0.09} \\
& =\sqrt{0.45} \\
\mathrm{I}_{\mathrm{T}} & =0.671 \mathrm{~A}
\end{aligned}
$$

3. Impedance

$$
\begin{aligned}
\mathrm{Z} & =\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{I}_{\mathrm{T}}} \\
& =\frac{120}{0.671} \\
\mathrm{Z} & =178.8 \Omega
\end{aligned}
$$

## Summary

Impedance is summarized below.

## Impedance Summary

- Impedance $(\mathrm{Z})$ is the total opposition to current flow in an AC circuit.
- The formula for impedance in a series AC circuit is:
$\mathrm{Z}=\sqrt{\mathrm{R}^{2}+\mathrm{X}^{2}}$
- The formula for impedance in a series R-C-L circuit is:

$$
\mathrm{Z}=\sqrt{\mathrm{R}^{2}+\left(\mathrm{X}_{\mathrm{C}}-\mathrm{X}_{\mathrm{L}}\right)^{2}}
$$

- The formulas for finding total current $\left(\mathrm{I}_{\mathrm{T}}\right)$ in a parallel R-C-L circuit are:
where $I_{C}>I_{L}, I_{T}=\sqrt{I_{R}^{2}+\left(I_{C}-I_{L}\right)^{2}}$
where $I_{L}>I_{C}, I_{T}=\sqrt{I_{R}^{2}+\left(I_{L}-I_{C}\right)^{2}}$


## RESONANCE

In the chapters on inductance and capacitance we have learned that both conditions are reactive and can provide opposition to current flow, but for opposite reasons. Therefore, it is important to find the point where inductance and capacitance cancel one another to achieve efficient operation of AC circuits.

EO 1.15 DEFINE resonance.
EO 1.16 Given the values of capacitance (C) and inductance ( L ), CALCULATE the resonant frequency.

EO 1.17 Given a series R-C-L circuit at resonance, DESCRIBE the net reactance of the circuit.

EO 1.18 Given a parallel R-C-L circuit at resonance, DESCRIBE the circuit output relative to current (I).

## Resonant Frequency

Resonance occurs in an AC circuit when inductive reactance and capacitive reactance are equal to one another: $\mathrm{X}_{\mathrm{L}}=\mathrm{X}_{\mathrm{C}}$. When this occurs, the total reactance, $\mathrm{X}=\mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{C}}$ becomes zero and the impendence is totally resistive. Because inductive reactance and capacitive reactance are both dependent on frequency, it is possible to bring a circuit to resonance by adjusting the frequency of the applied voltage. Resonant frequency $\left(f_{\text {Res }}\right)$ is the frequency at which resonance occurs, or where $X_{L}=X_{C}$. Equation (8-14) is the mathematical representation for resonant frequency.

$$
\begin{equation*}
\mathrm{f}_{\mathrm{Res}}=\frac{1}{2 \pi \sqrt{\mathrm{LC}}} \tag{8-14}
\end{equation*}
$$

where

$$
\begin{array}{ll}
\mathrm{f}_{\text {Res }} & =\text { resonant frequency }(\mathrm{Hz}) \\
\mathrm{L} & =\text { inductance }(\mathrm{H}) \\
\mathrm{C} & =\operatorname{capacitance}(\mathrm{f})
\end{array}
$$

## Series Resonance

In a series R-C-L circuit, as in Figure 9, at resonance the net reactance of the circuit is zero, and the impedance is equal to the circuit resistance; therefore, the current output of a series resonant circuit is at a maximum value for that circuit and is determined by the value of the resistance. ( $\mathrm{Z}=\mathrm{R}$ )

$$
\mathrm{I}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{Z}_{\mathrm{T}}}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{R}}
$$

## Parallel Resonance

Resonance in a parallel R-C-L circuit will occur when the reactive current in the inductive branches is equal to the reactive current in the capacitive branches (or when $X_{L}=X_{C}$ ). Because inductive and capacitive reactance currents are equal and opposite in phase, they cancel one another at parallel resonance.

If a capacitor and an inductor, each with negligible resistance, are connected in parallel and the frequency is adjusted such that reactances are exactly equal, current will flow in the inductor and the capacitor, but the total current will be negligible. The parallel C-L circuit will present an almost infinite impedance. The capacitor will alternately charge and discharge through the inductor. Thus, in a parallel R-C-L, as in Figure 10, the net current flow through the circuit is at minimum because of the high impendence presented by $X_{L}$ and $X_{C}$ in parallel.

## Summary

Resonance is summarized below.

## Resonance Summary

- Resonance is a state in which the inductive reactance equals the capacitive reactance ( $\mathrm{X}_{\mathrm{L}}=\mathrm{X}_{\mathrm{C}}$ ) at a specified frequency ( $\mathrm{f}_{\text {Res }}$ ).
- Resonant frequency is:

$$
\mathrm{f}_{\mathrm{Res}}=\frac{1}{2 \pi \sqrt{\mathrm{LC}}}
$$

- R-C-L series circuit at resonance is when net reactance is zero and circuit current output is determined by the series resistance of the circuit.
- R-C-L parallel circuit at resonance is when net reactance is maximum and circuit current output is at minimum.


## PART 3: BASIC AC POWER - TABLE OF CONTENTS

REFERENCES ..... ii
POWER TRIANGLE ..... 1
Power Triangle ..... 1
Apparent Power ..... 2
True Power ..... 3
Reactive Power ..... 3
Total Power ..... 4
Power Factor ..... 4
Power in Series R-L Circuit ..... 5
Power in Parallel R-L Circuit ..... 6
Power in Series R-C Circuit ..... 8
Power in Parallel R-C Circuit ..... 10
Power in Series R-C-L Circuit ..... 12
Power in Parallel R-C-L Circuit ..... 14
Summary ..... 16
THREE-PHASE CIRCUITS ..... 17
Three-Phase Systems ..... 17
Power in Balanced 3中 Loads ..... 19
Unbalanced 3 Loads ..... 23
Summary ..... 26

## REFERENCES

- Gussow, Milton, Schaum's Outline Series, Basic Electricity, McGraw-Hill.
- Academic Program for Nuclear Power Plant Personnel, Volume IV, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Academic Program for Nuclear Power Plant Personnel, Volume II, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Nasar and Unnewehr, Electromechanics and Electric Machines, John Wiley and Sons.
- Van Valkenburgh, Nooger, and Neville, Basic Electricity, Vol. 5, Hayden Book Company.
- Lister, Eugene C., Electric Circuits and Machines, $5^{\text {th }}$ Edition, McGraw-Hill.
- Croft, Carr, Watt, and Summers, American Electricians Handbook, $10^{\text {th }}$ Edition, McGraw-Hill.
- Mason, C. Russel, The Art and Science of Protective Relaying, John Wiley and Sons.
- Mileaf, Harry, Electricity One - Seven, Revised 2 ${ }^{\text {nd }}$ Edition, Hayden Book Company.
- Buban and Schmitt, Understanding Electricity and Electronics, $3{ }^{\text {rd }}$ Edition, McGrawHill.
- Kidwell, Walter, Electrical Instruments and Measurements, McGraw-Hill.


## POWER TRIANGLE

While direct current has one form of power, alternating current has three different forms of power that are related in a unique relationship. In this chapter, you will learn that power in AC circuits cannot be calculated in the same manner as in DC circuits.

EO 1.1 DESCRIBE the relationship between apparent, true, and reactive power by definition or by using a power triangle.

EO 1.2 DEFINE power factor as it relates to true power and apparent power.

EO 1.3 Given the necessary values for voltage (E), resistance $(\mathrm{R})$, reactance ( X ), impedance ( Z ), and/or current (I), CALCULATE the following power components for an AC circuit:
a. True power ( $\mathbf{P}$ )
b. Apparent power (S)
c. Reactive power (Q)
d. Power factor (pf)

EO 1.4 DEFINE the following terms:
a. Leading power factor
b. Lagging power factor

## Power Triangle

In AC circuits, current and voltage are normally out of phase and, as a result, not all the power produced by the generator can be used to accomplish work. By the same token, power cannot be calculated in AC circuits in the same manner as in DC circuits. The power triangle, shown in Figure 1, equates AC power to DC power by showing the relationship between generator output (apparent power - S ) in volt-amperes (VA), usable power (true power - P) in watts, and wasted or stored power (reactive power - Q) in volt-amperes-reactive (VAR). The phase angle $(\theta)$ represents the inefficiency of the AC circuit and corresponds to the total reactive impedance $(Z)$ to the current flow in the circuit.


Figure 1 Power Triangle

The power triangle represents comparable values that can be used directly to find the efficiency level of generated power to usable power, which is expressed as the power factor (discussed later). Apparent power, reactive power, and true power can be calculated by using the DC equivalent (RMS value) of the AC voltage and current components along with the power factor.

## Apparent Power

Apparent power ( S ) is the power delivered to an electrical circuit. Equation (9-1) is a mathematical representation of apparent power. The measurement of apparent power is in voltamperes (VA).

$$
\begin{equation*}
\mathrm{S}=\mathrm{I}^{2} \mathrm{Z}=\mathrm{I}_{\mathrm{T}} \mathrm{E} \tag{9-1}
\end{equation*}
$$

where
S = apparent power (VA)
I = RMS current (A)
$\mathrm{E}=\mathrm{RMS}$ voltage (V)
$\mathrm{Z}=$ impedance $(\Omega)$

## True Power

True power $(\mathrm{P})$ is the power consumed by the resistive loads in an electrical circuit. Equation $(9-2)$ is a mathematical representation of true power. The measurement of true power is in watts.

$$
\begin{equation*}
\mathrm{P}=\mathrm{I}^{2} \mathrm{R}=\mathrm{EI} \cos \theta \tag{9-2}
\end{equation*}
$$

where
P = true power (watts)
I = RMS current (A)
$\mathrm{E}=\mathrm{RMS}$ voltage (V)
$\mathrm{R}=$ resistance $(\Omega)$
$\theta=$ angle between E and I sine waves

## Reactive Power

Reactive power $(\mathrm{Q})$ is the power consumed in an AC circuit because of the expansion and collapse of magnetic (inductive) and electrostatic (capacitive) fields. Reactive power is expressed in volt-amperes-reactive (VAR). Equation (9-3) is a mathematical representation for reactive power.

$$
\begin{equation*}
\mathrm{Q}=\mathrm{I}^{2} \mathrm{X}=\mathrm{EI} \sin \theta \tag{9-3}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{Q}=\text { reactive power (VAR) } \\
& \mathrm{I}=\mathrm{RMS} \text { current }(\mathrm{A}) \\
& \mathrm{X}=\text { net reactance }(\Omega) \\
& \mathrm{E}=\mathrm{RMS} \text { voltage }(\mathrm{V}) \\
& \theta=\text { angle between the } \mathrm{E} \text { and I sine waves }
\end{aligned}
$$

Unlike true power, reactive power is not useful power because it is stored in the circuit itself. This power is stored by inductors, because they expand and collapse their magnetic fields in an attempt to keep current constant, and by capacitors, because they charge and discharge in an attempt to keep voltage constant. Circuit inductance and capacitance consume and give back reactive power. Reactive power is a function of a system's amperage. The power delivered to the inductance is stored in the magnetic field when the field is expanding and returned to the source when the field collapses. The power delivered to the capacitance is stored in the electrostatic field when the capacitor is charging and returned to the source when the capacitor discharges. None of the power delivered to the circuit by the source is consumed. It is all returned to the source. The true power, which is the power consumed, is thus zero. We know that alternating current constantly changes; thus, the cycle of expansion and collapse of the magnetic and electrostatic fields constantly occurs.

## Total Power

The total power delivered by the source is the apparent power. Part of this apparent power, called true power, is dissipated by the circuit resistance in the form of heat. The rest of the apparent power is returned to the source by the circuit inductance and capacitance.

## Power Factor

Power factor $(\mathrm{pf})$ is the ratio between true power and apparent power. True power is the power consumed by an AC circuit, and reactive power is the power that is stored in an AC circuit. $\operatorname{Cos} \theta$ is called the power factor (pf) of an AC circuit. It is the ratio of true power to apparent power, where $\theta$ is the phase angle between the applied voltage and current sine waves and also between P and S on a power triangle (Figure1). Equation (9-4) is a mathematical representation of power factor.

$$
\begin{equation*}
\cos \theta=\frac{\mathrm{P}}{\mathrm{~S}} \tag{9-4}
\end{equation*}
$$

where

$$
\begin{array}{lll}
\cos \theta & = & \text { power factor }(\mathrm{pf}) \\
\mathrm{P} & = & \text { true power }(\text { watts }) \\
\mathrm{S} & = & \text { apparent power }(\text { VA })
\end{array}
$$

Power factor also determines what part of the apparent power is real power. It can vary from 1 , when the phase angle is $0^{\circ}$, to 0 , when the phase angle is $90^{\circ}$. In an inductive circuit, the current lags the voltage and is said to have a lagging power factor, as shown in Figure 2.


Figure 3 Leading Power Factor


Figure 2 Lagging Power Factor

In a capacitive circuit, the current leads the voltage and is said to have a leading power factor, as shown in Figure 3.

A mnemonic memory device, "ELI the ICE man," can be used to remember the voltage/current relationship in AC circuits. ELI refers to an inductive circuit (L) where current (I) lags voltage (E). ICE refers to a capacitive circuit (C) where current (I) leads voltage (E).

## Power in Series R-L Circuit

Example: A $200 \Omega$ resistor and a $50 \Omega \mathrm{X}_{\mathrm{L}}$ are placed in series with a voltage source, and the total current flow is 2 amps , as shown in Figure 4.

Find: 1. pf
2. applied voltage, V
3. P
4. Q
5. S


Solution:
Figure 4 Series R-L Circuit

1. $\mathrm{pf}=\cos \theta \quad \theta=\arctan \left(\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}\right)$
$=\cos \left(\arctan \left(\frac{\mathrm{X}_{\mathrm{L}}}{\mathrm{R}}\right)\right)$
$=\cos \left(\arctan \left(\frac{50}{200}\right)\right)$
$=\cos \left(14^{\circ}\right)$
$\mathrm{pf}=0.097$
2. $\quad \mathrm{V}=\mathrm{IZ} \quad \mathrm{Z}=\sqrt{ } \mathrm{R}^{2}+\mathrm{X}_{\mathrm{L}}^{2}$
$=I / R^{2}+X_{L}^{2}$
$=2 \sqrt{ } 200^{2}+50^{2}$
$=2 \sqrt{ } 42,500$
$=(2)(206.16)$
$\mathrm{V}=412.3$ volts
Note: Inverse trigonometric functions such as arctan are discussed in the Mathematics Fundamentals Manual, Module 4, Trigonometry, pages 6 and 7 should the student require review.
3. $\mathrm{P}=\mathrm{EI} \cos \theta$
$=(412.3)(2)(0.97)$
$P=799.86$ watts
4. $\mathrm{Q}=\mathrm{EI} \sin \theta$
$=(412.3)(2)(0.242)$
$\mathrm{Q}=199.6 \mathrm{VAR}$
5. $\quad S=E I$
$=(412.3)(2)$
$\mathrm{S}=824.6 \mathrm{VA}$

## Power in Parallel R-L Circuit

Example: A $600 \Omega$ resistor and $200 \Omega \mathrm{X}_{\mathrm{L}}$ are in parallel with a 440 V source, as shown in Figure 5.

Find: 1. $\mathrm{I}_{\mathrm{T}}$
2. pf
3. P
4. Q
5. S


Figure 5 Parallel R-L Circuit

Solution:

1. $\mathrm{I}_{\mathrm{T}}=\sqrt{\mathrm{I}_{\mathrm{R}}^{2}+\mathrm{I}_{\mathrm{L}}^{2}} \quad \mathrm{I}_{\mathrm{R}}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{R}} \quad \mathrm{I}_{\mathrm{L}}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{X}_{\mathrm{L}}}$

$$
\begin{aligned}
& =\sqrt{\left(\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{R}}\right)^{2}+\left(\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{X}_{\mathrm{L}}}\right)^{2}} \\
& =\sqrt{\left(\frac{440}{600}\right)^{2}+\left(\frac{440}{200}\right)^{2}} \\
& =\sqrt{(0.73)^{2}+(2.2)^{2}} \\
& \mathrm{I}_{\mathrm{T}}
\end{aligned}=2.3 \mathrm{amps}
$$

2. $\mathrm{pf}=\cos \theta \quad \theta=\arctan \left(-\frac{\mathrm{I}_{\mathrm{L}}}{\mathrm{I}_{\mathrm{R}}}\right)$
$=\cos \left(\arctan \left(-\frac{I_{L}}{I_{R}}\right)\right)$
$=\cos \left(\arctan \left(-\frac{2.2}{0.73}\right)\right)$
$=\cos (\arctan (-3))$
$=\cos \left(-71.5^{\circ}\right)$
$\mathrm{pf}=0.32$
3. $\mathrm{P}=\mathrm{EI} \cos \theta$
$=(440)(2.3)(0.32)$
$\mathrm{P}=323.84$ watts
4. $\quad \mathrm{Q}=\mathrm{EI} \sin \theta$
$=(440)(2.3)(0.948)$
$\mathrm{Q}=959.4 \mathrm{VAR}$
5. $\mathrm{S}=\mathrm{EI}$
$=(440)(2.3)$
$\mathrm{S}=1012 \mathrm{VA}$

## Power in Series R-C Circuit

Example: An $80 \Omega X_{c}$ and a $60 \Omega$ resistance are in series with a 120V source, as shown in Figure 6.

Find: 1. Z
2. $\mathrm{I}_{\mathrm{T}}$
3. pf
4. P
5. Q
6. S

Solution:


Figure 6 Series R-C Circuit

1. $\mathrm{Z}=\sqrt{ } \mathrm{R}^{2}+\mathrm{X}_{\mathrm{C}}^{2}$
$=\sqrt{60^{2}}+80^{2}$
$=\sqrt{ } 3600+6400$
$Z=100 \Omega$
2. $\mathrm{I}_{\mathrm{T}}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{Z}}$

$$
=\frac{120}{100}
$$

$$
\mathrm{I}_{\mathrm{T}}=1.2 \mathrm{amps}
$$

3. $\mathrm{pf}=\cos \theta \quad \theta=\arctan \left(-\frac{\mathrm{X}_{\mathrm{C}}}{\mathrm{R}}\right)$
$=\cos \left(\arctan \left(-\frac{X_{C}}{R}\right)\right)$
$=\cos \left(\arctan \left(-\frac{80}{60}\right)\right)$
$=\cos (\arctan (-1.33))$
$=\cos \left(-53^{\circ}\right)$

$$
\mathrm{pf}=0.60
$$

4. $P=E I \cos \theta$
$=(120)(1.2)(0.60)$
$P=86.4$ watts
5. $\mathrm{Q}=\mathrm{EI} \sin \theta$
$=(120)(1.2)(0.798)$
$\mathrm{Q}=114.9 \mathrm{VAR}$
6. $\quad S=E I$
$=(120)(1.2)$
$S=144 \mathrm{VA}$

## Power in Parallel R-C Circuit

Example: A $30 \Omega$ resistance and a $40 \Omega \mathrm{X}_{\mathrm{C}}$ are in parallel with a 120 V power source, as shown in Figure 7.

Find: 1. $\mathrm{I}_{\mathrm{T}}$
2. Z
3. pf
4. P
5. Q
6. S


Solution:
Figure 7 Parallel R-C Circuit

1. $\mathrm{I}_{\mathrm{T}}=\sqrt{\mathrm{I}_{\mathrm{R}}^{2}+\mathrm{I}_{\mathrm{C}}^{2}} \quad \mathrm{I}_{\mathrm{R}}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{R}} \quad \mathrm{I}_{\mathrm{C}}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{X}_{\mathrm{C}}}$

$$
\begin{aligned}
& =\sqrt{\left(\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{R}}\right)^{2}+\left(\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{X}_{\mathrm{C}}}\right)^{2}} \\
& =\sqrt{\left(\frac{120}{30}\right)^{2}+\left(\frac{120}{40}\right)^{2}} \\
& =\sqrt{4^{2}+3^{2}}
\end{aligned}
$$

$$
\mathrm{I}_{\mathrm{T}}=5 \mathrm{amps}
$$

2. $\mathrm{Z}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{I}_{\mathrm{T}}}$

$$
=\frac{120}{5}
$$

$Z=24 \Omega$
3. $\mathrm{pf}=\cos \theta \quad \theta=\arctan \left(\frac{\mathrm{I}_{\mathrm{C}}}{\mathrm{I}_{\mathrm{R}}}\right)$
$=\cos \left(\arctan \left(\frac{I_{C}}{I_{R}}\right)\right)$
$=\cos \left(\arctan \left(\frac{3}{4}\right)\right)$
$=\cos \left(\arctan \left(36.9^{\circ}\right)\right)$
$\mathrm{pf}=0.80$
4. $\mathrm{P}=\mathrm{EI} \cos \theta$
$=(120)(5)(0.80)$
$\mathrm{P}=480$ watts
5. $\mathrm{Q}=\mathrm{EI} \sin \theta$
$=(120)(5)(0.6)$
$\mathrm{Q}=360 \mathrm{VAR}$
6. $\quad S=E I$
$=(120)(5)$
$\mathrm{S}=600 \mathrm{VA}$

## Power in Series R-C-L Circuit

Example: An $8 \Omega$ resistance, a $40 \Omega \mathrm{X}_{\mathrm{L}}$, and a $24 \Omega \mathrm{X}_{\mathrm{C}}$ are in series with a 60 Hz source with a current flow of 4 amps , as shown in Figure 8.

Find: 1. Z
2. $\mathrm{V}_{\mathrm{T}}$
3. pf
4. P
5. Q
6. S


Figure 8 Series R-C-L Circuit

1. $\mathrm{Z}=\sqrt{ } \mathrm{R}^{2}+\left(\mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{C}}\right)^{2}$
$=\sqrt{ } 8^{2}+(40-24)^{2}$
$=\sqrt{ } 8^{2}+16^{2}$
$Z=17.9 \Omega$
2. $V_{T}=I Z$
$=(4)(17.9)$
$\mathrm{V}_{\mathrm{T}}=71.6$ volts
3. $\mathrm{pf}=\cos \theta \quad \theta=\arctan \left(\frac{\mathrm{X}}{\mathrm{R}}\right)$
$=\cos \left(\arctan \left(\frac{X}{R}\right)\right)$
$=\cos \left(\arctan \left(\frac{16}{8}\right)\right)$
$=\cos (\arctan (2))$
$=\cos \left(63.4^{\circ}\right)$

$$
\mathrm{pf}=0.45
$$

4. $P=E I \cos \theta$
$=(71.6)(4)(0.45)$
$\mathrm{P}=128.9$ watts
5. $\mathrm{Q}=\mathrm{EI} \sin \theta$
$=(71.6)(4)(0.89)$
$\mathrm{Q}=254.9 \mathrm{VAR}$
6. $S=E I$
$=(71.6)(4)$
$\mathrm{S}=286.4 \mathrm{VA}$

## Power in Parallel R-C-L Circuits

Example: An $800 \Omega$ resistance, $100 \Omega \mathrm{X}_{\mathrm{L}}$, and an $80 \Omega \mathrm{X}_{\mathrm{C}}$ are in parallel with a $120 \mathrm{~V}, 60 \mathrm{~Hz}$ source, as shown in Figure 9.

Find: $1 . \mathrm{I}_{\mathrm{T}}$
2. pf
3. P
4. Q
5. S


Figure 9 Parallel R-C-L Circuit
Solution:

1. $\mathrm{I}_{\mathrm{T}}=\sqrt{\mathrm{I}_{\mathrm{R}}^{2}+\left(\mathrm{I}_{\mathrm{C}}-\mathrm{I}_{\mathrm{L}}\right)^{2}} \quad \mathrm{I}_{\mathrm{R}}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{R}} \quad \mathrm{I}_{\mathrm{L}}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{X}_{\mathrm{L}}} \quad \mathrm{I}_{\mathrm{C}}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{X}_{\mathrm{C}}}$

$$
\begin{aligned}
& =\sqrt{\left(\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{R}}\right)^{2}+\left(\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{X}_{\mathrm{L}}}-\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{X}_{\mathrm{C}}}\right)^{2}} \\
& =\sqrt{\left(\frac{120}{800}\right)^{2}+\left(\frac{120}{100}-\frac{120}{80}\right)^{2}} \\
& =\sqrt{0.15^{2}+(1.2-1.5)^{2}} \\
& =\sqrt{0.15^{2}+(-0.3)^{2}} \\
& \mathrm{I}_{\mathrm{T}}
\end{aligned}=0.34 \mathrm{amps}
$$

2. $\mathrm{pf}=\cos \theta \quad \theta=\arctan \left(\frac{\mathrm{I}_{\mathrm{C}}-\mathrm{I}_{\mathrm{L}}}{\mathrm{I}_{\mathrm{R}}}\right)$
$=\cos \left(\arctan \left(\frac{I_{C}-I_{L}}{I_{R}}\right)\right)$
$=\cos \left(\arctan \left(\frac{1.5-1.2}{0.15}\right)\right)$
$=\cos (\arctan (2))$
$=\cos \left(63.4^{\circ}\right)$

$$
\mathrm{pf}=0.45
$$

3. $P=E I \cos \theta$
$=(120)(0.34)(0.45)$
$\mathrm{P}=18.36$ watts
4. $\mathrm{Q}=\mathrm{EI} \sin \theta$
$=(120)(0.34)(0.89)$
$\mathrm{Q}=36.4 \mathrm{VAR}$
5. $\quad \mathrm{S}=\mathrm{EI}$
$=(120)(0.34)$
$\mathrm{S}=40.8 \mathrm{VA}$

## Summary

AC power relationships are summarized below.

## AC Power Relationships Summary

- Observe the equations for apparent, true, and reactive power, and power factor:
- Apparent power $(S)=I^{2} Z=I_{T} \mathrm{E}$
- $\quad$ True power $(\mathrm{P})=\mathrm{I}^{2} \mathrm{R}=$ EI $\cos \theta$
- $\quad$ Reactive power $(\mathrm{Q})=\mathrm{I}^{2} \mathrm{X}=\mathrm{EI} \sin \theta$
- Power factor $(\mathrm{pf})=\frac{\mathrm{P}}{\mathrm{S}}=\cos \theta$
- From observation, you can see that three power equations have the angle $\theta$ in common. $\theta$ is the angle between voltage and current. From this relationship, a power triangle, as shown in Figure 1, is formed.
- ELI the ICE man is a mnemonic device that describes the reactive characteristics of an AC circuit.
- $\quad$ Current (I) lags voltage (E) in an inductive circuit (L)
- $\quad$ Current (I) leads voltage (E) in a capacitive circuit (C)


## THREE-PHASE CIRCUITS

The design of three-phase AC circuits lends itself to a more efficient method of producing and utilizing an AC voltage.

EO 1.5 STATE the reasons that three-phase power systems are used in the industry.

EO 1.6 Given values for current, voltage, and power factor in a three-phase system, CALCULATE the following:
a. Real power
b. Reactive power
c. Apparent power

EO 1.7 Given a diagram of a wye- or delta-connected threephase system, DESCRIBE the voltage/current relationships of the circuit.

EO 1.8 STATE the indications of an unbalanced load in a threephase power system.

## Three-Phase Systems

A three-phase $(3 \phi)$ system is a combination of three single-phase systems. In a $3 \phi$ balanced system, power comes from a $3 \phi$ AC generator that produces three separate and equal voltages, each of which is $120^{\circ}$ out of phase with the other voltages (Figure 10).


Figure 10 Three-Phase AC

Three-phase equipment (motors, transformers, etc.) weighs less than single-phase equipment of the same power rating. They have a wide range of voltages and can be used for single-phase loads. Three-phase equipment is smaller in size, weighs less, and is more efficient than single-phase equipment.

Three-phase systems can be connected in two different ways. If the three common ends of each phase are connected at a common point and the other three ends are connected to a $3 \phi$ line, it is called a wye, or Y-, connection (Figure 11). If the three phases are connected in series to form a closed loop, it is called a delta, or $\Delta$-, connection.


Figure $113 \phi$ AC Power Connections

## Power in Balanced 3ф Loads

Balanced loads, in a $3 \phi$ system, have identical impedance in each secondary winding (Figure 12). The impedance of each winding in a delta load is shown as $\mathrm{Z}_{\Delta}$ (Figure 12a), and the impedence in a wye load is shown as $\mathrm{Z}_{\mathrm{y}}$ (Figure 12b). For either the delta or wye connection, the lines A, $B$, and C supply a $3 \phi$ system of voltages.

(a) Balanced $\Delta$ load, $Z_{A}=Z_{B}=Z_{C}=Z_{\Delta} \quad$ (b) Balanced $Y$ load, $Z_{A}=Z_{B}=Z_{C}=Z_{Y}$


Figure 12 3 ${ }^{\phi}$ Balanced Loads

In a balanced delta load, the line voltage $\left(\mathrm{V}_{\mathrm{L}}\right)$ is equal to the phase voltage $\left(\mathrm{V}_{\phi}\right)$, and the line current $\left(I_{L}\right)$ is equal to the square root of three times the phase current $\left(\sqrt{ } 3 I_{\phi}\right)$. Equation (9-5) is a mathematical representation of $V_{L}$ in a balanced delta load. Equation (9-6) is a mathematical representation of $\mathrm{I}_{\mathrm{L}}$ in a balanced delta load.

$$
\begin{align*}
& \mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\phi}  \tag{9-5}\\
& \mathrm{I}_{\mathrm{L}}=\sqrt{ } 3 \mathrm{I}_{\phi} \tag{9-6}
\end{align*}
$$

In a balanced wye load, the line voltage $\left(\mathrm{V}_{\mathrm{L}}\right)$ is equal to the square root of three times phase voltage $\left(\sqrt{ } 3 \mathrm{~V}_{\phi}\right)$, and line current $\left(I_{L}\right)$ is equal to the phase current $\left(I_{\phi}\right)$. Equation (9-7) is a mathematical representation of $\mathrm{V}_{\mathrm{L}}$ in a balanced wye load. Equation (9-8) is a mathematical representation of $I_{L}$ in a balanced wye load.

$$
\begin{align*}
& \mathrm{V}_{\mathrm{L}}=\sqrt{ } 3 \mathrm{~V}_{\phi}  \tag{9-7}\\
& \mathrm{I}_{\mathrm{L}}=\mathrm{I}_{\phi} \tag{9-8}
\end{align*}
$$

Because the impedance of each phase of a balanced delta or wye load has equal current, phase power is one third of the total power. Equation (9-10) is the mathematical representation for phase power $\left(\mathrm{P}_{\phi}\right)$ in a balanced delta or wye load.

$$
\begin{equation*}
P_{\phi}=V_{\phi} I_{\phi} \cos \theta \tag{9-10}
\end{equation*}
$$

Total power $\left(\mathrm{P}_{\mathrm{T}}\right)$ is equal to three times the single-phase power. Equation $(9-11)$ is the mathematical representation for total power in a balanced delta or wye load.

$$
\begin{equation*}
\mathrm{P}_{\mathrm{T}}=3 \mathrm{~V}_{\phi} \mathrm{I}_{\phi} \cos \theta \tag{9-11}
\end{equation*}
$$

In a delta-connected load, $V_{L}=V_{\phi}$ and $I_{\phi}=\frac{\sqrt{ } 3 I_{L}}{3}$ so:

$$
P_{T}=\sqrt{ } 3 V_{L} I_{L} \cos \theta
$$

In a wye-connected load, $I_{L}=I_{\phi}$ and $V_{\phi}=\frac{\sqrt{ } 3 V_{L}}{3}$ so:

$$
P_{T}=\sqrt{ } 3 V_{L} I_{L} \cos \theta
$$

As you can see, the total power formulas for delta- and wye-connected loads are identical.

Total apparent power $\left(\mathrm{S}_{\mathrm{T}}\right)$ in volt-amperes and total reactive power $\left(\mathrm{Q}_{\mathrm{T}}\right)$ in volt-amperes-reactive are related to total real power ( $\mathrm{P}_{\mathrm{T}}$ ) in watts (Figure 13).

A balanced three-phase load has the real, apparent, and reactive powers given by:

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{T}}=\sqrt{ } 3 \mathrm{~V}_{\mathrm{T}} \mathrm{I}_{\mathrm{L}} \cos \theta \\
& \mathrm{~S}_{\mathrm{T}}=\sqrt{ } 3 \mathrm{~V}_{\mathrm{T}} \mathrm{I}_{\mathrm{L}} \\
& \mathrm{Q}_{\mathrm{T}}=\sqrt{ } 3 \mathrm{~V}_{\mathrm{T}} \mathrm{I}_{\mathrm{L}} \sin \theta
\end{aligned}
$$



Figure $133 \phi$ Power Triangle

Example 1: Each phase of a deltaconnected 3中 AC generator supplies a full load current of 200 A at 440 volts with a 0.6 lagging power factor, as shown in Figure 14.

Find: 1. $\mathrm{V}_{\mathrm{L}}$
2. $\mathrm{I}_{\mathrm{L}}$
3. $\mathrm{P}_{\mathrm{T}}$
4. $Q_{T}$
5. $\mathrm{S}_{\mathrm{T}}$


Figure 14 Three-Phase Delta Generator

## Solution:

1. $\quad \mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\phi}$

$$
\mathrm{V}_{\mathrm{L}}=440 \text { volts }
$$

2. $I_{L}=\sqrt{ } 3 I_{\phi}$

$$
=(1.73)(200)
$$

$$
\mathrm{I}_{\mathrm{L}}=346 \mathrm{amps}
$$

3. $\mathrm{P}_{\mathrm{T}}=\sqrt{ } 3 \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \theta$
$=(1.73)(440)(346)(0.6)$
$\mathrm{P}_{\mathrm{T}}=158.2 \mathrm{~kW}$
4. $\quad \mathrm{Q}_{\mathrm{T}}=\sqrt{ } 3 \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \sin \theta$
$=(1.73)(440)(346)(0.8)$
$\mathrm{Q}_{\mathrm{T}}=210.7 \mathrm{kVAR}$
5. $\quad S_{T}=\sqrt{3} V_{L} I_{L}$
$=(1.73)(440)(346)$
$\mathrm{S}_{\mathrm{T}}=263.4 \mathrm{kVA}$
Example 2: Each phase of a wyeconnected $3 \phi$ AC generator supplies a 100 A current at a phase voltage of 240 V and a power factor of 0.9 lagging, as shown in Figure 15.

Find: 1. $\mathrm{V}_{\mathrm{L}}$
2. $\mathrm{P}_{\mathrm{T}}$
3. $Q_{T}$
4. $\mathrm{S}_{\mathrm{T}}$


Figure 15 Three-Phase Wye Generator

## Solution:

1. $\quad \mathrm{V}_{\mathrm{L}}=\sqrt{ } 3 \mathrm{~V}_{\phi}$
$=(1.73)(240)$
$\mathrm{V}_{\mathrm{L}}=415.2$ volts
2. $P_{T}=\sqrt{3} V_{L} I_{L} \cos \theta$
$=(1.73)(415.2)(100)(0.9)$

$$
\mathrm{P}_{\mathrm{T}}=64.6 \mathrm{~kW}
$$

3. $\mathrm{Q}_{\mathrm{T}}=\sqrt{ } 3 \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \sin \theta$
$=(1.73)(415.2)(100)(0.436)$ $\mathrm{Q}_{\mathrm{T}}=31.3 \mathrm{kVAR}$
4. $\quad S_{T}=\sqrt{3} V_{L} I_{L}$
$=(1.73)(415.2)(100)$

$$
\mathrm{S}_{\mathrm{T}}=71.8 \mathrm{kVA}
$$

## Unbalanced 3 $\phi$ Loads

An important property of a three-phase balanced system is that the phasor sum of the three line or phase voltages is zero, and the phasor sum of the three line or phase currents is zero. When the three load impedances are not equal to one another, the phasor sums and the neutral current $\left(I_{n}\right)$ are not zero, and the load is, therefore, unbalanced. The imbalance occurs when an open or short circuit appears at the load.

If a three-phase system has an unbalanced load and an unbalanced power source, the methods of fixing the system are complex. Therefore, we will only consider an unbalanced load with a balanced power source.

Example: A $3 \phi$ balanced system, as shown in Figure 16a, contains a wye load. The line-to- line voltage is 240 V , and the resistance is $40 \Omega$ in each branch.


Figure 16 3 $\phi$ Unbalanced Load

Find line current and neutral current for the following load conditions.

1. balanced load
2. open circuit phase A (Figure 16b)
3. short circuit in phase A (Figure 16c)
4. $I_{L}=I_{\phi} \quad I_{\phi}=\frac{V_{\phi}}{R_{\phi}} \quad V_{\phi}=\frac{V_{L}}{\sqrt{3}}$
$I_{L}=\frac{\left(\frac{\mathrm{V}_{\mathrm{L}}}{\sqrt{ } 3}\right)}{\mathrm{R}_{\phi}}$
$=\frac{\left(\frac{240}{1.73}\right)}{40}$
$=\frac{138.7}{40}$
$\mathrm{I}_{\mathrm{L}}=3.5 \mathrm{amps} \quad \mathrm{I}_{\mathrm{N}}=0$
5. Current flow in lines $B$ and $C$ becomes the resultant of the loads in $B$ and $C$ connected in series.

$$
\begin{aligned}
\mathrm{I}_{\mathrm{B}} & =\frac{\mathrm{V}_{\mathrm{L}}}{\mathrm{R}_{\mathrm{B}}+\mathrm{R}_{\mathrm{C}}} \quad \mathrm{I}_{\mathrm{C}}=\mathrm{I}_{\mathrm{B}} \\
& =\frac{240}{40+40} \\
\mathrm{I}_{\mathrm{B}} & =3 \mathrm{amps} \quad \mathrm{I}_{\mathrm{C}}=3 \mathrm{amps} \\
\mathrm{I}_{\mathrm{N}} & =\mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{C}} \\
& =3+3 \\
I_{N} & =6 \mathrm{amps}
\end{aligned}
$$

3. $I_{B}=\frac{V_{L}}{R_{B}} \quad I_{C}=I_{B}$

$$
=\frac{240}{40}
$$

$$
I_{B}=6 \mathrm{amps} \quad I_{C}=6 \mathrm{amps}
$$

The current in Phase $A$ is equal to the neutral line current, $I_{A}=I_{N}$. Therefore, $I_{N}$ is the phasor sum of $I_{B}$ and $I_{C}$.

$$
\begin{aligned}
\mathrm{I}_{\mathrm{N}} & =\sqrt{ } 3 \mathrm{I}_{\mathrm{B}} \\
& =(1.73)(6) \\
\mathrm{I}_{\mathrm{N}} & =10.4 \mathrm{amps}
\end{aligned}
$$

In a fault condition, the neutral connection in a wye-connected load will carry more current than the phase under a balanced load. Unbalanced three-phase circuits are indicated by abnormally high currents in one or more of the phases. This may cause damage to equipment if the imbalance is allowed to continue.

## Summary

Three-phase circuits are summarized below.

## Three-Phase Circuits Summary

- Three-phase power systems are used in the industry because:
- Three-phase circuits weigh less than single-phase circuits of the same power rating.
- They have a wide range of voltages and can be used for single-phase loads.
- Three-phase equipment is smaller in size, weighs less, and is more efficient than single-phase equipment.
- Unbalanced three-phase circuits are indicated by abnormally high currents in one or more of the phases.


## PART 4: AC GENERATORS - TABLE OF CONTENTS

REFERENCES ..... ii
AC GENERATOR COMPONENTS ..... 1
Field ..... 1
Armature ..... 1
Prime Mover ..... 1
Rotor ..... 2
Stator ..... 2
Slip Rings ..... 3
Summary ..... 4
AC GENERATOR THEORY ..... 5
Theory of Operation ..... 5
Losses in an AC Generator ..... 6
Hysteresis Losses ..... 7
Mechanical Losses ..... 7
Efficiency ..... 7
Summary ..... 8
AC GENERATOR OPERATION ..... 9
Ratings ..... 9
Paralleling AC Generators ..... 10
Types of AC Generators ..... 10
Three-Phase AC Generators ..... 11
AC Generator Connections ..... 12
Summary ..... 14

## REFERENCES

- Gussow, Milton, Schaum's Outline Series, Basic Electricity, McGraw-Hill.
- Academic Program for Nuclear Power Plant Personnel, Volume IV, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Academic Program for Nuclear Power Plant Personnel, Volume II, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Nasar and Unnewehr, Electromechanics and Electric Machines, John Wiley and Sons.
- Van Valkenburgh, Nooger, and Neville, Basic Electricity, Vol. 5, Hayden Book Company.
- Lister, Eugene C., Electric Circuits and Machines, $5^{\text {th }}$ Edition, McGraw-Hill.
- Croft, Carr, Watt, and Summers, American Electricians Handbook, $10^{\text {th }}$ Edition, McGrawHill.
- Mason, C. Russel, The Art and Science of Protective Relaying, John Wiley and Sons.
- Mileaf, Harry, Electricity One - Seven, Revised $2^{\text {nd }}$ Edition, Hayden Book Company.
- Buban and Schmitt, Understanding Electricity and Electronics, $3^{\text {rd }}$ Edition, McGraw-Hill.
- Kidwell, Walter, Electrical Instruments and Measurements, McGraw-Hill.


## AC GENERATOR COMPONENTS

AC generators are widely used to produce AC voltage. To understand how these generators operate, the function of each component of the generator must first be understood.

EO 1.1 STATE the purpose of the following components of an
AC generator:
a. Field
b. Armature
c. Prime mover
d. Rotor
e. Stator
f. Slip rings

## Field

The field in an AC generator consists of coils of conductors within the generator that receive a voltage from a source (called excitation) and produce a magnetic flux. The magnetic flux in the field cuts the armature to produce a voltage. This voltage is ultimately the output voltage of the AC generator.

## Armature

The armature is the part of an AC generator in which voltage is produced. This component consists of many coils of wire that are large enough to carry the full-load current of the generator.

## Prime Mover

The prime mover is the component that is used to drive the AC generator. The prime mover may be any type of rotating machine, such as a diesel engine, a steam turbine, or a motor.

## Rotor

The rotor of an AC generator is the rotating component of the generator, as shown in Figure 1. The rotor is driven by the generator's prime mover, which may be a steam turbine, gas turbine, or diesel engine. Depending on the type of generator, this component may be the armature or the field. The rotor will be the armature if the voltage output is generated there; the rotor will be the field if the field excitation is applied there.


Figure 1 Basic AC Generator

## Stator

The stator of an AC generator is the part that is stationary (refer to Figure 1). Like the rotor, this component may be the armature or the field, depending on the type of generator. The stator will be the armature if the voltage output is generated there; the stator will be the field if the field excitation is applied there.

## Slip Rings

Slip rings are electrical connections that are used to transfer power to and from the rotor of an AC generator (refer to Figure 1). The slip ring consists of a circular conducting material that is connected to the rotor windings and insulated from the shaft. Brushes ride on the slip ring as the rotor rotates. The electrical connection to the rotor is made by connections to the brushes.

Slip rings are used in AC generators because the desired output of the generator is a sine wave. In a DC generator, a commutator was used to provide an output whose current always flowed in the positive direction, as shown in Figure 2. This is not necessary for an AC generator. Therefore, an AC generator may use slip rings, which will allow the output current and voltage to oscillate through positive and negative values. This oscillation of voltage and current takes the shape of a sine wave.


Figure 2-Comparison of DC and AC Generator Outputs

## Summary

The important information in this chapter is summarized below.

## AC Generator Components Summary

- The field in an AC generator consists of coils of conductors within the generator that receive a voltage from a source (called excitation) and produce a magnetic flux.
- The armature is the part of an AC generator in which output voltage is produced.
- The prime mover is the component that is used to drive the AC generator.
- The rotor of an AC generator is the part that is driven by the prime mover and that rotates.
- The stator of an AC generator is the part that is stationary.
- Slip rings are electrical connections that are used to transfer power to and from the rotor of an AC generator.


## AC GENERATOR THEORY

AC generators are widely used to produce AC voltage. To understand how these generators operate, the basic theory of operation must first be understood.

EO 1.2 Given the speed of rotation and number of poles, CALCULATE the frequency output of an AC generator.

EO 1.3 LIST the three losses found in an AC generator.
EO 1.4 Given the prime mover input and generator output, DETERMINE the efficiency of an AC generator.

## Theory of Operation

A simple AC generator consists of: (a) a strong magnetic field, (b) conductors that rotate through that magnetic field, and (c) a means by which a continuous connection is provided to the conductors as they are rotating (Figure 3). The strong magnetic field is produced by a current flow through the field coil of the rotor. The field coil in the rotor receives excitation through the use of slip rings and brushes. Two brushes are spring-held in contact with the slip rings to provide the continuous connection


Figure 3 Simple AC Generator between the field coil and the external excitation circuit. The armature is contained within the windings of the stator and is connected to the output. Each time the rotor makes one complete revolution, one complete cycle of AC is developed. A generator has many turns of wire wound into the slots of the rotor.

The magnitude of AC voltage generated by an AC generator is dependent on the field strength and speed of the rotor. Most generators are operated at a constant speed; therefore, the generated voltage depends on field excitation, or strength.

The frequency of the generated voltage is dependent on the number of field poles and the speed at which the generator is operated, as indicated in Equation (10-1).

$$
\begin{equation*}
\mathrm{f}=\frac{\mathrm{NP}}{120} \tag{10-1}
\end{equation*}
$$

where

$$
\begin{array}{rll}
\mathrm{f} & = & \text { frequency }(\mathrm{Hz}) \\
\mathrm{P} & = & \text { total number of poles } \\
\mathrm{N} & = & \text { rotor speed (rpm) } \\
120 & = & \text { conversion from minutes to seconds and from poles to pole pairs }
\end{array}
$$

The 120 in Equation (10-1) is derived by multiplying the following conversion factors.

$$
\frac{60 \text { seconds }}{1 \text { minute }} \times \frac{2 \text { poles }}{\text { pole pair }}
$$

In this manner, the units of frequency (hertz or cycles/sec.) are derived.

## Losses in an AC Generator

The load current flows through the armature in all AC generators. Like any coil, the armature has some amount of resistance and inductive reactance. The combination of these make up what is known as the internal resistance, which causes a loss in an AC generator. When the load current flows, a voltage drop is developed across the internal resistance. This voltage drop subtracts from the output voltage and, therefore, represents generated voltage and power that is lost and not available to the load. The voltage drop in an AC generator can be found using Equation (10-2).

$$
\begin{equation*}
\text { Voltage drop }=I_{a} R_{a}+I_{a} X_{L a} \tag{10-2}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{a}}=\text { armature current } \\
& \mathrm{R}_{\mathrm{a}}=\text { armature resistance } \\
& \mathrm{X}_{\mathrm{La}}=\text { armature inductive reactance }
\end{aligned}
$$

## Hysteresis Losses

Hysteresis losses occur when iron cores in an AC generator are subject to effects from a magnetic field. The magnetic domains of the cores are held in alignment with the field in varying numbers, dependent upon field strength. The magnetic domains rotate, with respect to the domains not held in alignment, one complete turn during each rotation of the rotor. This rotation of magnetic domains in the iron causes friction and heat. The heat produced by this friction is called magnetic hysteresis loss.

To reduce hysteresis losses, most AC armatures are constructed of heat-treated silicon steel, which has an inherently low hysteresis loss. After the heat-treated silicon steel is formed to the desired shape, the laminations are heated to a dull red and then allowed to cool. This process, known as annealing, reduces hysteresis losses to a very low value.

## Mechanical Losses

Rotational or mechanical losses can be caused by bearing friction, brush friction on the commutator, and air friction (called windage), which is caused by the air turbulence due to armature rotation. Careful maintenance can be instrumental in keeping bearing friction to a minimum. Clean bearings and proper lubrication are essential to the reduction of bearing friction. Brush friction is reduced by ensuring: proper brush seating, proper brush use, and maintenance of proper brush tension. A smooth and clean commutator also aids in the reduction of brush friction. In very large generators, hydrogen is used within the generator for cooling; hydrogen, being less dense than air, causes less windage losses than air.

## Efficiency

Efficiency of an AC generator is the ratio of the useful power output to the total power input. Because any mechanical process experiences some losses, no AC generators can be 100 percent efficient. Efficiency of an AC generator can be calculated using Equation (10-3).

$$
\begin{equation*}
\text { Efficiency }=\frac{\text { Output }}{\text { Input }} \times 100 \tag{10-3}
\end{equation*}
$$

Example: Given a 5 hp motor acting as the prime mover of a generator that has a load demand of 2 kW , what is the efficiency of the generator?

Solution:
In order to calculate efficiency, the input and output power must be in the same units. As described in Thermodynamics, the horsepower and the watt are equivalent units of power.

Therefore, the equivalence of these units is expressed with a conversion factor as follows.

$$
\begin{aligned}
& \left(\frac{550 \frac{\mathrm{ft}-\mathrm{lbf}}{\mathrm{sec}}}{1 \mathrm{hp}}\right)\left(\frac{1 \mathrm{~kW}}{737.6 \frac{\mathrm{ft}-\mathrm{lbf}}{\mathrm{sec}}}\right)\left(\frac{1000 \mathrm{w}}{1 \mathrm{~kW}}\right)=746 \frac{\mathrm{~W}}{\mathrm{hp}} \\
& \text { Input Power }=5 \mathrm{hp} \times 746 \frac{\mathrm{~W}}{\mathrm{hp}}=3730 \mathrm{~W} \\
& \text { Output Power }=2 \mathrm{~kW}=2000 \mathrm{~W} \\
& \text { Efficiency } \quad=\frac{\text { Output }}{\text { Input }}=\frac{2000 \mathrm{~W}}{3730 \mathrm{~W}}=0.54 \times 100=54 \%
\end{aligned}
$$

## Summary

The important information covered in this chapter is summarized below.

## AC Generator Theory Summary

- The frequency of the generated voltage in an AC generator can be calculated by multiplying the number of poles by the speed of the generator and dividing by a factor of 120 .
- The three losses found in an AC generator are:
- Internal voltage drops due to the internal resistance and impedance of the generator
- Hysteresis losses
- Mechanical losses
- Efficiency of an AC generator can be calculated by dividing the output by the input and multiplying by 100 .


## AC GENERATOR OPERATION

Because of the nature of $A C$ voltage and current, the operation of an $A C$ generator requires that rules and procedures be followed. In addition, there are various types of AC generators available, each type having advantages and disadvantages.

EO 1.5 DESCRIBE the bases behind the kW and current ratings of an AC generator.

EO 1.6 DESCRIBE the conditions that must be met prior to paralleling two AC generators including consequences of not meeting these conditions.

EO 1.7 DESCRIBE the difference between a stationary field, rotating armature AC generator and a rotating field, stationary armature AC generator.

EO 1.8 EXPLAIN the differences between a wye-connected and delta-connected AC generator including advantages and disadvantages of each type.

## Ratings

Typical name plate data for an AC generator (Figure 4) includes: (1) manufacturer; (2) serial number and type number; (3) speed (rpm), number of poles, frequency of output, number of phases, and maximum supply voltage; (4) capacity rating in KVA and kW at a specified power factor and maximum output voltage; (5) armature and field current per phase; and (6) maximum temperature rise.

Power (kW) ratings of an AC generator are based on the ability of the prime mover to overcome generator losses and the ability of the machine to dissipate the internally generated heat. The current rating of an AC generator is based on the insulation rating of the machine.


Figure 4 AC Generator Nameplate Ratings

## Paralleling AC Generators

Most electrical power grids and distribution systems have more than one AC generator operating at one time. Normally, two or more generators are operated in parallel in order to increase the available power. Three conditions must be met prior to paralleling (or synchronizing) AC generators.

- Their terminal voltages must be equal. If the voltages of the two AC generators are not equal, one of the AC generators could be picked up as a reactive load to the other AC generator. This causes high currents to be exchanged between the two machines, possibly causing generator or distribution system damage.
- Their frequencies must be equal. A mismatch in frequencies of the two AC generators will cause the generator with the lower frequency to be picked up as a load on the other generator (a condition referred to as "motoring"). This can cause an overload in the generators and the distribution system.
- Their output voltages must be in phase. A mismatch in the phases will cause large opposing voltages to be developed. The worst case mismatch would be $180^{\circ}$ out of phase, resulting in an opposing voltage between the two generators of twice the output voltage. This high voltage can cause damage to the generators and distribution system due to high currents.

During paralleling operations, voltages of the two generators that are to be paralleled are indicated through the use of voltmeters. Frequency matching is accomplished through the use of output frequency meters. Phase matching is accomplished through the use of a synchroscope, a device that senses the two frequencies and gives an indication of phase differences and a relative comparison of frequency differences.

## Types of AC Generators

As previously discussed, there are two types of AC generators: the stationary field, rotating armature; and the rotating field, stationary armature.

Small AC generators usually have a stationary field and a rotating armature (Figure 5). One important disadvantage to this arrangement is that the slip-ring and brush assembly is in series with the load circuits and, because of worn or dirty components, may interrupt the flow of current.


Figure 5 Stationary Field, Rotating Armature AC Generator

If DC field excitation is connected to the rotor, the stationary coils will have AC induced into them (Figure 6). This arrangement is called a rotating field, stationary armature AC generator.

The rotating field, stationary armature type AC generator is used when large power generation is involved. In this type of generator, a DC source is supplied to the rotating field coils, which produces a magnetic field around the rotating element. As the rotor is turned by the prime mover, the


Figure 6 Simple AC Generator - Rotating Field, Stationary Armature field will cut the conductors of the stationary armature, and an EMF will be induced into the armature windings.

This type of AC generator has several advantages over the stationary field, rotating armature AC generator: (1) a load can be connected to the armature without moving contacts in the circuit; (2) it is much easier to insulate stator fields than rotating fields; and (3) much higher voltages and currents can be generated.

## Three-Phase AC Generators

The principles of a three-phase generator are basically the same as that of a single-phase generator, except that there are three equally-spaced windings and three output voltages that are all $120^{\circ}$ out of phase with one another. Physically adjacent loops (Figure 7) are separated by $60^{\circ}$ of rotation; however, the loops are connected to the slip rings in such a manner that there are 120 electrical degrees between phases.

The individual coils of each winding are combined and represented as a single coil. The significance of Figure 7 is that it shows that the three-phase generator has three separate armature windings that are 120 electrical degrees out of phase.


Figure 7 Stationary Armature $3 \phi$ Generator

## AC Generator Connections

As shown in Figure 7, there are six leads from the armature of a three-phase generator, and the output is connected to an external load. In actual practice, the windings are connected together, and only three leads are brought out and connected to the external load.

Two means are available to connect the three armature windings. In one type of connection, the windings are connected in series, or delta-connected ( $\Delta$ ) (Figure 8).

In a delta-connected generator, the voltage between any


Figure 8 Delta Connection two of the phases, called line voltage, is the same as the voltage generated in any one phase. As shown in Figure 9, the three phase voltages are equal, as are the three line voltages. The current in any line is $\sqrt{ } 3$ times the phase current. You can see that a delta-connected generator provides an increase in current, but no increase in voltage.


Figure 9 Characteristics of a Delta-Connected Generator

An advantage of the delta-connected AC generator is that if one phase becomes damaged or open, the remaining two phases can still deliver three-phase power. The capacity of the generator is reduced to $57.7 \%$ of what it was with all three phases in operation.

In the other type of connection, one of the leads of each winding is connected, and the remaining three leads are connected to an external load. This is called a wye connection (Y) (Figure 10).

The voltage and current characteristics of the wye-connected AC generator are opposite to that of the delta connection. Voltage between any two lines in a wyeconnected AC generator is 1.73 (or $\sqrt{ } 3$ ) times any one phase voltage, while line


Figure 10 Wye Connection currents are equal to phase currents. The wye-connected AC generator provides an increase in voltage, but no increase in current (Figure 11).


Figure 11 Characteristics of a Wye-Connected AC Generator

An advantage of a wye-connected AC generator is that each phase only has to carry $57.7 \%$ of line voltage and, therefore, can be used for high voltage generation.

## Summary

The important information covered in this chapter is summarized below.

## AC Generator Operation Summary

- Power ( kW ) ratings of an AC generator are based on the ability of the prime mover to overcome generation losses and the ability of the machine to dissipate the heat generated internally. The current rating of an AC generator is based on the insulation rating of the machine.
- There are three requirements that must be met to parallel AC generators:

1) Their terminal voltages must be equal. A mismatch may cause high currents and generator or distribution system damage.
2) Their frequencies must be equal. A mismatch in frequencies can cause one generator to "motor," causing an overload in the generators and the distribution system.
3) Their output voltages must be in phase. A mismatch in the phases will cause large opposing voltages to be developed, resulting in damage to the generators and distribution system due to high currents.

- The disadvantage of a stationary field, rotating armature is that the slip-ring and brush assembly is in series with the load circuits and, because of worn or dirty components, may interrupt the flow of current.
- A stationary armature, rotating field generator has several advantages: (1) a load can be connected to the armature without moving contacts in the circuit; (2) it is much easier to insulate stator fields than rotating fields; and (3) much higher voltages and currents can be generated.
- The advantage of the delta-connected AC generator is that if one phase becomes damaged or open, the remaining two phases can still deliver three-phase power at a reduced capacity of $57.7 \%$.
- The advantage of a wye-connected AC generator is that each phase only has to carry $57.7 \%$ of line voltage and, therefore, can be used for high voltage generation.


## PART 5: VOLTAGE REGULATORS - TABLE OF CONTENTS

REFERENCES ..... ii
VOLTAGE REGULATORS ..... 1
Purpose ..... 1
Block Diagram Description ..... 1
Sensing Circuit ..... 2
Reference Circuit ..... 2
Comparison Circuit ..... 2
Amplification Circuit ..... 2
Signal Output Circuit ..... 3
Feedback Circuit ..... 3
Changing Output Voltage ..... 3
Summary ..... 4

## REFERENCES

- Gussow, Milton, Schaum's Outline Series, Basic Electricity, McGraw-Hill.
- Academic Program for Nuclear Power Plant Personnel, Volume IV, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Academic Program for Nuclear Power Plant Personnel, Volume II, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Nasar and Unnewehr, Electromechanics and Electric Machines, John Wiley and Sons.
- Van Valkenburgh, Nooger, and Neville, Basic Electricity, Vol. 5, Hayden Book Company.
- Lister, Eugene C., Electric Circuits and Machines, $5^{\text {th }}$ Edition, McGraw-Hill.
- Croft, Carr, Watt, and Summers, American Electricians Handbook, $10^{\text {th }}$ Edition, McGrawHill.
- Mileaf, Harry, Electricity One - Seven, Revised $2^{\text {nd }}$ Edition, Hayden Book Company.
- Buban and Schmitt, Understanding Electricity and Electronics, $3^{\text {rd }}$ Edition, McGraw-Hill.
- Kidwell, Walter, Electrical Instruments and Measurements, McGraw-Hill.


## VOLTAGE REGULATORS

Because the voltage from an AC generator varies as the output load and power factor change, a voltage regulator circuit is necessary to permit continuity of the desired output voltage.

EO 1.1 STATE the purpose for voltage regulation equipment.
EO 1.2 Given a block diagram of a typical voltage regulator, DESCRIBE the function of each of the following components:
a. Sensing circuit
b. Reference circuit
c. Comparison circuit
d. Amplification circuit(s)
e. Signal output circuit
f. Feedback circuit

## Purpose

The purpose of a voltage regulator is to maintain the output voltage of a generator at a desired value. As load on an AC generator changes, the voltage will also tend to change. The main reason for this change in voltage is the change in the voltage drop across the armature winding caused by a change in load current. In an AC generator, there is an IR drop and an $\mathrm{IX}_{\mathrm{L}}$ drop caused by the AC current flowing through the resistance and inductance of the windings. The IR drop is dependent on the amount of the load change only. The $\mathrm{IX}_{\mathrm{L}}$ drop is dependent on not only the load change, but also the power factor of the circuit. Therefore, the output voltage of an AC generator varies with both changes in load (i.e., current) and changes in power factor. Because of changes in voltage, due to changes in load and changes in power factor, AC generators require some auxiliary means of regulating output voltage.

## Block Diagram Description

Figure 1 shows a typical block diagram of an AC generator voltage regulator. This regulator consists of six basic circuits that together regulate the output voltage of an AC generator from no-load to full-load.


Figure 1 Voltage Regulator Block Diagram

## Sensing Circuit

The sensing circuit senses output voltage of the AC generator. As the generator is loaded or unloaded, the output voltage changes, and the sensing circuit provides a signal of these voltage changes. This signal is proportional to output voltage and is sent to the comparison circuit.

## Reference Circuit

The reference circuit maintains a constant output for reference. This reference is the desired voltage output of the AC generator.

## Comparison Circuit

The comparison circuit electrically compares the reference voltage to the sensed voltage and provides an error signal. This error signal represents an increase or decrease in output voltage. The signal is sent to the amplification circuit.

## Amplification Circuit

The amplification circuit, which can be a magnetic amplifier or transistor amplifier, takes the signal from the comparison circuit and amplifies the milliamp input to an amp output, which is then sent to the signal output, or field, circuit.

## Signal Output Circuit

The signal output circuit, which controls field excitation of the AC generator, increases or decreases field excitation to either raise or lower the AC output voltage.

## Feedback Circuit

The feedback circuit takes some of the output of the signal output circuit and feeds it back to the amplification circuit. It does this to prevent overshooting or undershooting of the desired voltage by slowing down the circuit response.

## Changing Output Voltage

Let us consider an increase in generator load and, thereby, a drop in output voltage. First, the sensing circuit senses the decrease in output voltage as compared to the reference and lowers its input to the comparison circuit. Since the reference circuit is always a constant, the comparison circuit will develop an error signal due to the difference between the sensed voltage and the reference voltage. The error signal developed will be of a positive value with the magnitude of the signal dependent on the difference between the sensed voltage and the reference voltage. This output from the comparison circuit will then be amplified by the amplifier circuit and sent to the signal output circuit. The signal output circuit then increases field excitation to the AC generator. This increase in field excitation causes generated voltage to increase to the desired output.

If the load on the generator were decreased, the voltage output of the machine would rise. The actions of the voltage regulator would then be the opposite of that for a lowering output voltage. In this case, the comparison circuit will develop a negative error signal whose magnitude is again dependent on the difference between the sensed voltage and the reference voltage. As a result, the signal output circuit will decrease field excitation to the AC generator, causing the generated voltage to decrease to the desired output.

## Summary

Voltage regulators are summarized below.

## Voltage Regulators Summary

- Purpose - to maintain the output voltage of a generator at a desired value
- Sensing circuit - senses output voltage of the AC generator
- Reference circuit - maintains a constant output for reference, or desired, voltage output of the AC generator
- Comparison circuit - compares reference voltage to output voltage and provides an error signal to the amplification circuit
- Amplification circuit(s) - takes the signal from the comparison circuit and amplifies the milliamp input to an amp output
- Signal output circuit - controls field excitation of the AC generator
- Feedback circuit - prevents overshooting or undershooting of the desired voltage by slowing down the circuit response


## PART 6: AC MOTORS - TABLE OF CONTENTS

REFERENCES ..... ii
AC MOTOR THEORY ..... 1
Principles of Operation ..... 1
Rotating Field ..... 1
Torque Production ..... 5
Slip ..... 5
Torque ..... 7
Summary ..... 8
AC MOTOR TYPES ..... 9
Induction Motor ..... 9
Single-Phase AC Induction Motors ..... 11
Synchronous Motors ..... 12
Starting a Synchronous Motor ..... 12
Field Excitation ..... 14
Summary ..... 15

## REFERENCES

- Gussow, Milton, Schaum's Outline Series, Basic Electricity, McGraw-Hill.
- Academic Program for Nuclear Power Plant Personnel, Volume IV, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Academic Program for Nuclear Power Plant Personnel, Volume II, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Nasar and Unnewehr, Electromechanics and Electric Machines, John Wiley and Sons.
- Van Valkenburgh, Nooger, and Neville, Basic Electricity, Vol. 5, Hayden Book Company.
- Lister, Eugene C., Electric Circuits and Machines, $5^{\text {th }}$ Edition, McGraw-Hill.
- Croft, Carr, Watt, and Summers, American Electricians Handbook, $10^{\text {th }}$ Edition, McGrawHill.
- Mason, C. Russel, The Art and Science of Protective Relaying, John Wiley and Sons.
- Mileaf, Harry, Electricity One - Seven, Revised 2 ${ }^{\text {nd }}$ Edition, Hayden Book Company.
- Buban and Schmitt, Understanding Electricity and Electronics, $3^{\text {rd }}$ Edition, McGraw-Hill.
- Kidwell, Walter, Electrical Instruments and Measurements, McGraw-Hill.


## AC MOTOR THEORY

AC motors are widely used to drive machinery for a wide variety of applications. To understand how these motors operate, a knowledge of the basic theory of operation of AC motors is necessary.

EO 1.1 DESCRIBE how a rotating magnetic field is produced in an AC motor.

EO 1.2 DESCRIBE how torque is produced in an AC motor.
EO 1.3 Given field speed and rotor speed, CALCULATE percent slip in an AC motor.

EO 1.4 EXPLAIN the relationship between slip and torque in an AC induction motor.

## Principles of Operation

The principle of operation for all AC motors relies on the interaction of a revolving magnetic field created in the stator by AC current, with an opposing magnetic field either induced on the rotor or provided by a separate DC current source. The resulting interaction produces usable torque, which can be coupled to desired loads throughout the facility in a convenient manner. Prior to the discussion of specific types of AC motors, some common terms and principles must be introduced.

## Rotating Field

Before discussing how a rotating magnetic field will cause a motor rotor to turn, we must first find out how a rotating magnetic field is produced. Figure 1 illustrates a three-phase stator to which a three-phase AC current is supplied.

The windings are connected in wye. The two windings in each phase are wound in the same direction. At any instant in time, the magnetic field generated by one particular phase will depend on the current through that phase. If the current through that phase is zero, the resulting magnetic field is zero. If the current is at a maximum value, the resulting field is at a maximum value. Since the currents in the three windings are $120^{\circ}$ out of phase, the magnetic fields produced will also be $120^{\circ}$ out of phase. The three magnetic fields will combine to produce one field, which will act upon the rotor. In an AC induction motor, a magnetic field is induced in the rotor opposite in polarity of the magnetic field in the stator. Therefore, as the magnetic field rotates in the stator, the rotor also rotates to maintain its alignment with the stator's magnetic field. The remainder of this chapter's discussion deals with AC induction motors.


Figure 1 Three-Phase Stator

From one instant to the next, the magnetic fields of each phase combine to produce a magnetic field whose position shifts through a certain angle. At the end of one cycle of alternating current, the magnetic field will have shifted through $360^{\circ}$, or one revolution (Figure 2). Since the rotor has an opposing magnetic field induced upon it, it will also rotate through one revolution.

For purpose of explanation, rotation of the magnetic field is developed in Figure 2 by "stopping" the field at six selected positions, or instances. These instances are marked off at $60^{\circ}$ intervals on the sine waves representing the current flowing in the three phases, A, B, and C. For the following discussion, when the current flow in a phase is positive, the magnetic field will develop a north pole at the poles labeled $\mathrm{A}, \mathrm{B}$, and C . When the current flow in a phase is negative, the magnetic field will develop a north pole at the poles labeled A', B', and C'.


Figure 2 Rotating Magnetic Field

At point T1, the current in phase C is at its maximum positive value. At the same instance, the currents in phases A and B are at half of the maximum negative value. The resulting magnetic field is established vertically downward, with the maximum field strength developed across the C phase, between pole C (north) and pole $\mathrm{C}^{\prime}$ (south). This magnetic field is aided by the weaker fields developed across phases A and B, with poles A' and B' being north poles and poles A and $B$ being south poles.

At Point T2, the current sine waves have rotated through 60 electrical degrees. At this point, the current in phase A has increased to its maximum negative value. The current in phase B has reversed direction and is at half of the maximum positive value. Likewise, the current in phase C has decreased to half of the maximum positive value. The resulting magnetic field is established downward to the left, with the maximum field strength developed across the A phase, between poles A' (north) and A (south). This magnetic field is aided by the weaker fields developed across phases $B$ and $C$, with poles $B$ and $C$ being north poles and poles $B^{\prime}$ and $C$ ' being south poles. Thus, it can be seen that the magnetic field within the stator of the motor has physically rotated $60^{\circ}$.

At Point T3, the current sine waves have again rotated 60 electrical degrees from the previous point for a total rotation of 120 electrical degrees. At this point, the current in phase B has increased to its maximum positive value. The current in phase A has decreased to half of its maximum negative value, while the current in phase C has reversed direction and is at half of its maximum negative value also. The resulting magnetic field is established upward to the left, with the maximum field strength developed across phase B , between poles B (north) and B ' (south). This magnetic field is aided by the weaker fields developed across phases A and C, with poles A' and C' being north poles and poles A and C being south poles. Thus, it can be seen that the magnetic field on the stator has rotated another $60^{\circ}$ for a total rotation of $120^{\circ}$.

At Point T4, the current sine waves have rotated 180 electrical degrees from Point T1 so that the relationship of the phase currents is identical to Point T 1 except that the polarity has reversed. Since phase C is again at a maximum value, the resulting magnetic field developed across phase C will be of maximum field strength. However, with current flow reversed in phase C the magnetic field is established vertically upward between poles C' (north) and C (south). As can be seen, the magnetic field has now physically rotated a total of $180^{\circ}$ from the start.

At Point T5, phase A is at its maximum positive value, which establishes a magnetic field upward to the right. Again, the magnetic field has physically rotated $60^{\circ}$ from the previous point for a total rotation of $240^{\circ}$. At Point T6, phase B is at its maximum negative value, which will establish a magnetic field downward to the right. The magnetic field has again rotated $60^{\circ}$ from Point T5 for a total rotation of $300^{\circ}$.

Finally, at Point T7, the current is returned to the same polarity and values as that of Point T1. Therefore, the magnetic field established at this instance will be identical to that established at Point T1. From this discussion it can be seen that for one complete revolution of the electrical sine wave $\left(360^{\circ}\right)$, the magnetic field developed in the stator of a motor has also rotated one complete revolution $\left(360^{\circ}\right)$. Thus, you can see that by applying three-phase AC to three windings symmetrically spaced around a stator, a rotating magnetic field is generated.

## Torque Production

When alternating current is applied to the stator windings of an AC induction motor, a rotating magnetic field is developed. The rotating magnetic field cuts the bars of the rotor and induces a current in them due to generator action. The direction of this current flow can be found using the left-hand rule for generators. This induced current will produce a magnetic field, opposite in polarity of the stator field, around the conductors of the rotor, which will try to line up with the magnetic field of the stator. Since the stator field is rotating continuously, the rotor cannot line up with, or lock onto, the stator field and, therefore, must follow behind it (Figure 3).


Figure 3 Induction Motor

## Slip

It is virtually impossible for the rotor of an AC induction motor to turn at the same speed as that of the rotating magnetic field. If the speed of the rotor were the same as that of the stator, no relative motion between them would exist, and there would be no induced EMF in the rotor. (Recall from earlier modules that relative motion between a conductor and a magnetic field is needed to induce a current.) Without this induced EMF, there would be no interaction of fields to produce motion. The rotor must, therefore, rotate at some speed less than that of the stator if relative motion is to exist between the two.

The percentage difference between the speed of the rotor and the speed of the rotating magnetic field is called slip. The smaller the percentage, the closer the rotor speed is to the rotating magnetic field speed. Percent slip can be found by using Equation (12-1).

$$
\begin{equation*}
\mathrm{SLIP}=\frac{\mathrm{N}_{\mathrm{S}}-\mathrm{N}_{\mathrm{R}}}{\mathrm{~N}_{\mathrm{S}}} \times 100 \% \tag{12-1}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{N}_{\mathrm{S}}=\text { synchronous speed }(\mathrm{rpm}) \\
& \mathrm{N}_{\mathrm{R}}=\text { rotor speed }(\mathrm{rpm})
\end{aligned}
$$

The speed of the rotating magnetic field or synchronous speed of a motor can be found by using Equation (12-2).

$$
\begin{equation*}
\mathrm{N}_{\mathrm{S}}=\frac{120 \mathrm{f}}{\mathrm{P}} \tag{12-2}
\end{equation*}
$$

where
$\mathrm{N}_{\mathrm{s}}=$ speed of rotating field (rpm)
$\mathrm{f}=$ frequency of rotor current $(\mathrm{Hz})$
$\mathrm{P}=$ total number of poles
Example: A two pole, 60 Hz AC induction motor has a full load speed of 3554 rpm . What is the percent slip at full load?

Solution:
Synchronous speed:

$$
\begin{aligned}
& \mathrm{N}_{\mathrm{S}}=\frac{120 \mathrm{f}}{\mathrm{P}} \\
& \mathrm{~N}_{\mathrm{S}}=\frac{120(60 \mathrm{~Hz})}{2} \\
& \mathrm{~N}_{\mathrm{S}}=3600 \mathrm{rpm}
\end{aligned}
$$

Slip:

$$
\begin{aligned}
& \text { SLIP }=\frac{\mathrm{N}_{\mathrm{s}}-\mathrm{N}_{\mathrm{R}}}{\mathrm{~N}_{\mathrm{S}}} \times 100 \% \\
& \text { SLIP }=\frac{3600-3554 \mathrm{rpm}}{3600 \mathrm{rpm}} \times 100 \%=1.3 \%
\end{aligned}
$$

## Torque

The torque of an AC induction motor is dependent upon the strength of the interacting rotor and stator fields and the phase relationship between them. Torque can be calculated by using Equation (12-3).

$$
\begin{equation*}
\mathrm{T}=\mathrm{K} \Phi \mathrm{I}_{\mathrm{R}} \cos \theta_{\mathrm{R}} \tag{12-3}
\end{equation*}
$$

where

$$
\begin{array}{ll}
\mathrm{T} & =\text { torque (lb-ft) } \\
\mathrm{K} & =\text { constant } \\
\Phi & =\text { stator magnetic flux } \\
\mathrm{I}_{\mathrm{R}} & =\text { rotor current (A) } \\
\cos \theta_{\mathrm{R}} & =\text { power factor of rotor }
\end{array}
$$

During normal operation, $\mathrm{K}, \Phi$, and $\cos \theta_{\mathrm{R}}$ are, for all intents and purposes, constant, so that torque is directly proportional to the rotor current. Rotor current increases in almost direct proportion to slip. The change in torque with respect to slip (Figure 4) shows that, as slip increases from zero to $\sim 10 \%$, the torque increases linearly. As the load and slip are increased beyond full-load torque, the torque will reach a maximum value at about $25 \%$ slip. The maximum value of torque is called the breakdown torque of the motor. If load is increased beyond this point, the motor will stall and come to a rapid stop. The typical induction motor breakdown torque varies from 200 to $300 \%$ of full load torque. Starting


Figure 4 Torque vs Slip torque is the value of torque at $100 \%$ slip and is normally 150 to $200 \%$ of full-load torque. As the rotor accelerates, torque will increase to breakdown torque and then decrease to the value required to carry the load on the motor at a constant speed, usually between $0-10 \%$.

## Summary

The important information covered in this chapter is summarized below.

## AC Motor Theory Summary

- A magnetic field is produced in an AC motor through the action of the threephase voltage that is applied. Each of the three phases is $120^{\circ}$ from the other phases. From one instant to the next, the magnetic fields combine to produce a magnetic field whose position shifts through a certain angle. At the end of one cycle of alternating current, the magnetic field will have shifted through $360^{\circ}$, or one revolution.
- Torque in an AC motor is developed through interactions with the rotor and the rotating magnetic field. The rotating magnetic field cuts the bars of the rotor and induces a current in them due to generator action. This induced current will produce a magnetic field around the conductors of the rotor, which will try to line up with the magnetic field of the stator.
- Slip is the percentage difference between the speed of the rotor and the speed of the rotating magnetic field.
- In an AC induction motor, as slip increases from zero to $\sim 10 \%$, the torque increases linearly. As the load and slip are increased beyond full-load torque, the torque will reach a maximum value at about $25 \%$ slip. If load is increased beyond this point, the motor will stall and come to a rapid stop. The typical induction motor breakdown torque varies from 200 to $300 \%$ of full-load torque. Starting torque is the value of torque at $100 \%$ slip and is normally 150 to $200 \%$ of full-load torque.


## AC MOTOR TYPES

Various types of AC motors are used for specific applications. By matching the type of motor to the appropriate application, increased equipment performance can be obtained.

EO 1.5 DESCRIBE how torque is produced in a single-phase AC motor.

EO 1.6 EXPLAIN why an AC synchronous motor does not have starting torque.

EO 1.7 DESCRIBE how an AC synchronous motor is started.
EO 1.8 DESCRIBE the effects of over and under-exciting an AC synchronous motor.

EO 1.9 STATE the applications of the following types of AC motors:
a. Induction
b. Single-phase
c. Synchronous

## Induction Motor

Previous explanations of the operation of an AC motor dealt with induction motors. The induction motor is the most commonly used AC motor in industrial applications because of its simplicity, rugged construction, and relatively low manufacturing costs. The reason that the induction motor has these characteristics is because the rotor is a self-contained unit, with no external connections. This type of motor derives its name from the fact that AC currents are induced into the rotor by a rotating magnetic field.

The induction motor rotor (Figure 5) is made of a laminated cylinder with slots in its surface. The windings in the slots are one of two types. The most commonly used is the "squirrel-cage" rotor. This rotor is made of heavy copper bars that are connected at each end by a metal ring made of copper or brass. No insulation is required between the core and the bars because of the low voltages induced into the rotor bars. The size of the air gap between the rotor bars and stator windings necessary to obtain the maximum field strength is small.


Figure 5 Squirrel-Cage Induction Rotor


Figure 6 Split-Phase Motor

## Single-Phase AC Induction Motors

If two stator windings of unequal impedance are spaced 90 electrical degrees apart and connected in parallel to a single-phase source, the field produced will appear to rotate. This is called phase splitting.

In a split-phase motor, a starting winding is utilized. This winding has a higher resistance and lower reactance than the main winding (Figure 6). When the same voltage $\mathrm{V}_{\mathrm{T}}$ is applied to the starting and main windings, the current in the main winding ( $\mathrm{I}_{\mathrm{M}}$ ) lags behind the current of the starting winding $I_{S}$ (Figure 6). The angle between the two windings is enough phase difference to provide a rotating magnetic field to produce a starting torque. When the motor reaches 70 to $80 \%$ of synchronous speed, a centrifugal switch on the motor shaft opens and disconnects the starting winding.

Single-phase motors are used for very small commercial applications such as household appliances and buffers.


Figure 7 Wound Rotor

## Synchronous Motors

Synchronous motors are like induction motors in that they both have stator windings that produce a rotating magnetic field. Unlike an induction motor, the synchronous motor is excited by an external DC source and, therefore, requires slip rings and brushes to provide current to the rotor. In the synchronous motor, the rotor locks into step with the rotating magnetic field and rotates at synchronous speed. If the synchronous motor is loaded to the point where the rotor is pulled out of step with the rotating magnetic field, no torque is developed, and the motor will stop. A synchronous motor is not a self-starting motor because torque is only developed when running at synchronous speed; therefore, the motor needs some type of device to bring the rotor to synchronous speed.

Synchronous motors use a wound rotor. This type of rotor contains coils of wire placed in the rotor slots. Slip rings and brushes are used to supply current to the rotor. (Figure 7).

## Starting a Synchronous Motor

A synchronous motor may be started by a DC motor on a common shaft. When the motor is brought to synchronous speed, AC current is applied to the stator windings. The DC motor now acts as a DC generator and supplies DC field excitation to the rotor of the synchronous motor. The load may now be placed on the synchronous motor. Synchronous motors are more often started by means of a squirrel-cage winding embedded in the face of the rotor poles. The motor is then started as an induction motor and brought to $\sim 95 \%$ of synchronous speed, at which time direct current is applied, and the motor begins to pull into synchronism. The torque required to pull the motor into synchronism is called the pull-in torque.

As we already know, the synchronous motor rotor is locked into step with the rotating magnetic field and must continue to operate at synchronous speed for all loads. During no-load conditions, the center lines of a pole of the rotating magnetic field and the DC field pole coincide (Figure 8a). As load is applied to the motor, there is a backward shift of the rotor pole, relative to the stator pole (Figure 8b). There is no change in speed. The angle between the rotor and stator poles is called the torque angle ( $\alpha$ ).


Figure 8 Torque Angle
If the mechanical load on the motor is increased to the point where the rotor is pulled out of synchronism $\left(\alpha \cong 90^{\circ}\right)$, the motor will stop. The maximum value of torque that a motor can develop without losing synchronism is called its pull-out torque.

## Field Excitation

For a constant load, the power factor of a synchronous motor can be varied from a leading value to a lagging value by adjusting the DC field excitation (Figure 9). Field excitation can be adjusted so that $\mathrm{PF}=1$ (Figure 9a). With a constant load on the motor, when the field excitation is increased, the counter EMF $\left(\mathrm{V}_{\mathrm{G}}\right)$ increases. The result is a change in phase between stator current (I) and terminal voltage $\left(\mathrm{V}_{\mathrm{t}}\right)$, so that the motor operates at a leading power factor (Figure $9 b) . V_{p}$ in Figure 9 is the voltage drop in the stator winding's due to the impedance of the windings and is $90^{\circ}$ out of phase with the stator current. If we reduce field excitation, the motor will operate at a lagging power factor (Figure 9c). Note that torque angle, $\alpha$, also varies as field excitation is adjusted to change power factor.


Figure 9 Synchronous Motor Field Excitation

Synchronous motors are used to accommodate large loads and to improve the power factor of transformers in large industrial complexes.

## Summary

The important information in this chapter is summarized below.

## AC Motor Types Summary

- In a split-phase motor, a starting winding is utilized. This winding has a higher resistance and lower reactance than the main winding. When the same voltage $\left(\mathrm{V}_{\mathrm{T}}\right)$ is applied to the starting and main windings, the current in the main winding lags behind the current of the starting winding. The angle between the two windings is enough phase difference to provide a rotating magnetic field to produce a starting torque.
- A synchronous motor is not a self-starting motor because torque is only developed when running at synchronous speed.
- A synchronous motor may be started by a DC motor on a common shaft or by a squirrel-cage winding imbedded in the face of the rotor poles.
- Keeping the same load, when the field excitation is increased on a synchronous motor, the motor operates at a leading power factor. If we reduce field excitation, the motor will operate at a lagging power factor.
- The induction motor is the most commonly used AC motor in industrial applications because of its simplicity, rugged construction, and relatively low manufacturing costs.
- Single-phase motors are used for very small commercial applications such as household appliances and buffers.
- Synchronous motors are used to accommodate large loads and to improve the power factor of transformers in large industrial complexes.
REFERENCES ..... ii
TRANSFORMER THEORY ..... 1
Mutual Induction ..... 1
Turns Ratio ..... 2
Impedance Ratio ..... 2
Efficiency ..... 3
Theory of Operation ..... 3
Voltage Ratio ..... 5
Current Ratio ..... 7
Three-Phase Transformer Connections ..... 8
Delta Connection ..... 8
Wye Connection ..... 8
Combinations of Delta and Wye Transformer Connections ..... 9
Transformer Losses and Efficiency ..... 11
Transformer Operation Under No-Load ..... 13
Coil Polarity ..... 14
Summary ..... 16
TRANSFORMER TYPES ..... 17
Types of Transformers ..... 17
Distribution Transformer ..... 17
Power Transformer ..... 17
Control Transformer ..... 18
Auto Transformer ..... 18
Isolation Transformer ..... 18
Instrument Potential Transformer ..... 19
Instrument Current Transformer ..... 19
Summary ..... 20


## REFERENCES

- Gussow, Milton, Schaum's Outline Series, Basic Electricity, McGraw-Hill.
- Academic Program for Nuclear Power Plant Personnel, Volume IV, Columbia, MD: General Physics Corporation, Library of Congress Card \#A 326517, 1982.
- Nasar and Unnewehr, Electromechanics and Electric Machines, John Wiley and Sons.
- Van Valkenburgh, Nooger, and Neville, Basic Electricity, Vol. 5, Hayden Book Company.
- Croft, Carr, Watt, and Summers, American Electricians Handbook, $10^{\text {th }}$ Edition, McGrawHill.
- Mileaf, Harry, Electricity One - Seven, Revised $2^{\text {nd }}$ Edition, Hayden Book Company.
- Buban and Schmitt, Understanding Electricity and Electronics, $3^{\text {rd }}$ Edition, McGraw-Hill.


## TRANSFORMER THEORY

Transformers are used extensively for AC power transmissions and for various control and indication circuits. Knowledge of the basic theory of how these components operate is necessary to understand the role transformers play in today's nuclear facilities.

EO 1.1 DEFINE the following terms as they pertain to transformers:
a. Mutual induction
b. Turns ratio
c. Impedance ratio
d. Efficiency

EO 1.2 DESCRIBE the differences between a wye-connected and delta-connected transformer.

EO 1.3 Given the type of connection and turns ratios for the primary and secondary of a transformer, CALCULATE voltage, current, and power for each of the following types:
a. $\Delta-\Delta$
b. $\Delta-\mathrm{Y}$
c. $\quad \mathbf{Y}-\Delta$
d. $\quad Y-Y$

## Mutual Induction

If flux lines from the expanding and contracting magnetic field of one coil cut the windings of another nearby coil, a voltage will be induced in that coil. The inducing of an EMF in a coil by magnetic flux lines generated in another coil is called mutual induction. The amount of electromotive force (EMF) that is induced depends on the relative positions of the two coils.

## Turns Ratio

Each winding of a transformer contains a certain number of turns of wire. The turns ratio is defined as the ratio of turns of wire in the primary winding to the number of turns of wire in the secondary winding. Turns ratio can be expressed using Equation (13-1).

$$
\begin{equation*}
\text { Turns ratio }=\frac{N_{P}}{N_{S}} \tag{13-1}
\end{equation*}
$$

where

$$
\mathrm{N}_{\mathrm{P}}=\text { number of turns on the primary coil }
$$

$\mathrm{N}_{\mathrm{S}}=$ number of turns on the secondary coil
The coil of a transformer that is energized from an AC source is called the primary winding (coil), and the coil that delivers this AC to the load is called the secondary winding (coil) (Figure 1).

## Impedance Ratio

Maximum power is transferred from one circuit to another through a transformer when the impedances are equal, or matched. A transformer winding constructed with a definite turns ratio can perform an impedance matching function. The turns ratio will establish the proper relationship between the primary and secondary winding impedances. The ratio between the two impedances is referred to as the impedance ratio and is expressed by using Equation (13-2).

$$
\begin{equation*}
\left(\frac{\mathrm{N}_{\mathrm{P}}}{\mathrm{~N}_{\mathrm{S}}}\right)^{2}=\frac{\mathrm{Z}_{\mathrm{p}}}{\mathrm{Z}_{\mathrm{S}}} \tag{13-2}
\end{equation*}
$$

Another way to express the impedance ratio is to take the square root of both sides of Equation (13-2). This puts the ratio in terms of the turns ratio, which is always given for a transformer.

$$
\frac{\mathrm{N}_{\mathrm{P}}}{\mathrm{~N}_{\mathrm{S}}}=\sqrt{\frac{\mathrm{Z}_{\mathrm{P}}}{\mathrm{Z}_{\mathrm{S}}}}
$$

where
$\mathrm{N}_{\mathrm{P}}=$ number of turns in the primary
$\mathrm{N}_{\mathrm{S}}=$ number of turns in the secondary
$\mathrm{Z}_{\mathrm{P}}=$ impedance of primary
$\mathrm{Z}_{\mathrm{S}}=$ impedance of secondary

## Efficiency

Efficiency of a transformer is the ratio of the power output to the power input, as illustrated by Equation (13-3).

$$
\begin{equation*}
\text { Efficiency }=\frac{\text { Power Output }}{\text { Power Input }}=\frac{\mathrm{P}_{\mathrm{S}}}{\mathrm{P}_{\mathrm{P}}} \times 100 \tag{13-3}
\end{equation*}
$$

where
$P_{S}=$ power of secondary
$P_{P}=$ power of primary

## Theory of Operation

A transformer works on the principle that energy can be transferred by magnetic induction from one set of coils to another set by means of a varying magnetic flux. The magnetic flux is produced by an AC source.

The coil of a transformer that is energized from an AC source is called the primary winding (coil), and the coil that delivers this AC to the load is called the secondary winding (coil) (Figure $1)$.

In Figure 1, the primary and secondary coils are shown on separate legs of the magnetic circuit so that we can easily understand how the transformer works. Actually, half of the primary and secondary coils are wound on each of the two legs, with sufficient insulation between the two coils and the core to properly insulate the windings from one another and the core. A transformer wound, such as in Figure 1, will operate at a greatly reduced efficiency due to the magnetic leakage. Magnetic leakage is the part of the magnetic flux that passes through either one of the coils, but not through both. The larger the distance between the primary and secondary windings, the longer the magnetic circuit and the greater the leakage.


Figure 1 Core-Type Transformer

When alternating voltage is applied to the primary winding, an alternating current will flow that will magnetize the magnetic core, first in one direction and then in the other direction. This alternating flux flowing around the entire length of the magnetic circuit induces a voltage in both the primary and secondary windings. Since both windings are linked by the same flux, the voltage induced per turn of the primary and secondary windings must be the same value and same direction. This voltage opposes the voltage applied to the primary winding and is called counter-electromotive force (CEMF).

## Voltage Ratio

The voltage of the windings in a transformer is directly proportional to the number of turns on the coils. This relationship is expressed in Equation (13-4).

$$
\begin{equation*}
\frac{\mathrm{V}_{\mathrm{P}}}{\mathrm{~V}_{\mathrm{S}}}=\frac{\mathrm{N}_{\mathrm{P}}}{\mathrm{~N}_{\mathrm{S}}} \tag{13-4}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{P}}=\text { voltage on primary coil } \\
& \mathrm{V}_{\mathrm{S}}=\text { voltage on secondary coil } \\
& \mathrm{N}_{\mathrm{P}}=\text { number of turns on the primary coil } \\
& \mathrm{N}_{\mathrm{S}}=\text { number of turns on the secondary coil }
\end{aligned}
$$

The ratio of primary voltage to secondary voltage is known as the voltage ratio (VR). As mentioned previously, the ratio of primary turns of wire to secondary turns of wire is known as the turns ratio (TR). By substituting into the Equation (13-4), we find that the voltage ratio is equal to the turns ratio.

$$
\mathrm{VR}=\mathrm{TR}
$$

A voltage ratio of $1: 5$ means that for each volt on the primary, there will be 5 volts on the secondary. If the secondary voltage of a transformer is greater than the primary voltage, the transformer is referred to as a "step-up" transformer. A ratio of 5:1 means that for every 5 volts on the primary, there will only be 1 volt on the secondary. When secondary voltage is less than primary voltage, the transformer is referred to as a "step-down" transformer.

Example 1: A transformer (Figure 2) reduces voltage from 120 volts in the primary to 6 volts in the secondary. If the primary winding has 300 turns and the secondary has 15 turns, find the voltage and turns ratio.

Solution:
$\mathrm{VR}=\frac{\mathrm{V}_{\mathrm{P}}}{\mathrm{V}_{\mathrm{S}}}=\frac{120}{60}=\frac{20}{1}=20: 1$
$\mathrm{TR}=\frac{\mathrm{N}_{\mathrm{P}}}{\mathrm{N}_{\mathrm{S}}}=\frac{300}{15}=\frac{20}{1}=20: 1$


Figure 2 Example 1 Transformer

Example 2: An iron core transformer with a primary voltage of 240 volts has 250 turns in the primary and 50 turns in the secondary. Find the secondary voltage.

Solution:

$$
\frac{\mathrm{V}_{\mathrm{P}}}{\mathrm{~V}_{\mathrm{S}}}=\frac{\mathrm{N}_{\mathrm{P}}}{\mathrm{~N}_{\mathrm{S}}}
$$

Next, solve for $V_{S}$.

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{S}}=\frac{\mathrm{N}_{\mathrm{S}}}{\mathrm{~N}_{\mathrm{P}}} \mathrm{~V}_{\mathrm{P}} \\
& \mathrm{~V}_{\mathrm{S}}=\frac{50}{250} 240 \text { volts } \\
& \mathrm{V}_{\mathrm{S}}=48 \text { volts }
\end{aligned}
$$

Example 3: A power transformer has a turns ratio of 1:4. If the secondary coil has 5000 turns and secondary voltage is 60 volts, find the voltage ratio, $V_{P}$, and $\mathrm{N}_{\mathrm{P}}$.

Solution:

$$
\begin{aligned}
& \mathrm{VR}=\mathrm{TR} \\
& \mathrm{VR}=1: 4 \\
& \frac{\mathrm{~V}_{\mathrm{P}}}{\mathrm{~V}_{\mathrm{S}}}=\mathrm{VR}=1: 4=\frac{1}{4} \\
& \mathrm{~V}_{\mathrm{P}}=\frac{1}{4} \mathrm{~V}_{\mathrm{S}}=\frac{60}{4}=15 \text { volts } \\
& \mathrm{TR}=\frac{\mathrm{N}_{\mathrm{P}}}{\mathrm{~N}_{\mathrm{S}}}=\frac{1}{4} \\
& \mathrm{~N}_{\mathrm{P}}=\frac{1}{4} \mathrm{~N}_{\mathrm{S}}=\frac{5000}{4}=1250 \text { turns }
\end{aligned}
$$

## Current Ratio

The current in the windings of a transformer is inversely proportional to the voltage in the windings. This relationship is expressed in Equation (13-5).

$$
\begin{equation*}
\frac{\mathrm{V}_{\mathrm{P}}}{\mathrm{~V}_{\mathrm{S}}}=\frac{\mathrm{I}_{\mathrm{S}}}{\mathrm{I}_{\mathrm{P}}} \tag{13-5}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{P}}=\text { primary coil current } \\
& \mathrm{I}_{\mathrm{S}}=\text { secondary coil current }
\end{aligned}
$$

Since the voltage ratio is equal to the turns ratio, we can express the current ratio in terms of the turns ratio, as in Equation (13-6).

$$
\begin{equation*}
\frac{\mathrm{N}_{\mathrm{P}}}{\mathrm{~N}_{\mathrm{S}}}=\frac{\mathrm{I}_{\mathrm{S}}}{\mathrm{I}_{\mathrm{P}}} \tag{13-6}
\end{equation*}
$$

Example 1: When operated at 120 V in the primary of an iron core transformer, the current in the primary is 4 amps . Find the current in the secondary if the voltage is stepped up to 500 V .

Solution:

$$
\frac{\mathrm{V}_{\mathrm{P}}}{\mathrm{~V}_{\mathrm{S}}}=\frac{\mathrm{I}_{\mathrm{S}}}{\mathrm{I}_{\mathrm{P}}}
$$

Next, we solve for $I_{S}$.

$$
\begin{aligned}
& I_{S}=\frac{V_{P}}{V_{S}} I_{P} \\
& I_{S}=\frac{120}{500} 4 \mathrm{amps} \\
& I_{S}=0.96 \mathrm{amps}
\end{aligned}
$$

Example 2: A transformer with 480 turns on the primary and 60 turns on the secondary draws 0.6 amps from a 120 V line. Find $\mathrm{I}_{\mathrm{s}}$.

Solution:

$$
\frac{\mathrm{N}_{\mathrm{P}}}{\mathrm{~N}_{\mathrm{S}}}=\frac{\mathrm{I}_{\mathrm{S}}}{\mathrm{I}_{\mathrm{P}}}
$$

Next, we solve for $I_{S}$.

$$
\begin{aligned}
& I_{S}=\frac{N_{P}}{N_{S}} I_{P} \\
& I_{S}=\frac{480}{60} 0.6 \mathrm{amps} \\
& I_{S}=4.8 \mathrm{amps}
\end{aligned}
$$

The student should note from the previous examples that a transformer that "steps-up" voltage, "steps-down" the current proportionally.

## Three-Phase Transformer Connections

So far, our discussion has dealt with the operation of single-phase transformers. Three-phase transformer operation is identical except that three single-phase windings are used. These windings may be connected in wye, delta, or any combination of the two.

## Delta Connection

In the delta connection, all three phases are connected in series to form a closed loop
(Figure $3)$.


Figure 3 Delta Connection

## Wye Connection

In the wye connection, three common ends of each phase are connected together at a common terminal (marked " N " for neutral), and the other three ends are connected to a three-phase line (Figure 4).


Figure 4 Wye Connection

## Combinations of Delta and Wye Transformer Connections

A three-phase transformer may have three separate but identical single-phase ( $1 \phi$ ) transformers or a single $3 \phi$ unit containing three-phase windings. The transformer windings may be connected to form a $3 \phi$ bank in any of four different ways (Figure 5).


Figure $53 \phi$ Transformer Connections

Figure 5 shows the voltages and currents in terms of applied line voltage (V) and line current (I), where the turns ratio (a) is equal to one. Voltage and current ratings of the individual transformers depend on the connections (Figure 5) and are indicated by Table 1 for convenience of calculations.

| Transformer Connection (Primary to Secondary) | Primary |  |  |  | Secondary |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Line |  | Phase |  | Line |  | Phase |  |
|  | Volt. | Current | Volt. | Current | Volt. | Current | Volt. | Current |
| $\Delta-\Delta$ | V | I | V | $\frac{\mathrm{I}}{\sqrt{3}}$ | $\frac{\mathrm{V}}{\mathrm{a}}$ | aI | $\frac{\mathrm{V}}{\mathrm{a}}$ | $\frac{\mathrm{aI}}{\sqrt{3}}$ |
| Y-Y | V | I | $\frac{\mathrm{V}}{\sqrt{3}}$ | I | $\frac{\mathrm{V}}{\mathrm{a}}$ | aI | $\frac{\mathrm{V}}{\sqrt{3 \mathrm{a}}}$ | aI |
| Y- $\Delta$ | V | I | $\frac{\mathrm{V}}{\sqrt{3}}$ | I | $\frac{\mathrm{V}}{\sqrt{3} \mathrm{a}}$ | $\sqrt{3} \mathrm{aI}$ | $\frac{\mathrm{V}}{\sqrt{3} \mathrm{a}}$ | aI |
| $\Delta$-Y | V | I | V | $\frac{\mathrm{I}}{\sqrt{3}}$ | $\frac{\sqrt{3} \mathrm{~V}}{\mathrm{a}}$ | $\frac{\mathrm{aI}}{\sqrt{3}}$ | $\frac{\mathrm{V}}{\mathrm{a}}$ | $\frac{\mathrm{aI}}{\sqrt{3}}$ |

$*_{\mathrm{a}}=\mathrm{N}_{1} / \mathrm{N}_{2} ; \sqrt{3}=1.73$
Example 1: If line voltage is 440 V to a $3 \phi$ transformer bank, find the voltage across each primary winding for all four types of transformer connections.
$\Delta-\Delta:$ primary voltage $=\mathrm{V}=440$ volts
Y-Y: primary voltage $=\frac{\mathrm{V}}{\sqrt{3}}=\frac{440}{1.73}=254.3$ volts
$\mathrm{Y}-\Delta:$ primary voltage $=\frac{\mathrm{V}}{\sqrt{3}}=\frac{440}{1.73}=254.3$ volts
$\Delta-\mathrm{Y}$ : primary voltage $=\mathrm{V}=440$ volts
Example 2: If line current is 10.4 A in a $3 \phi$ transformer connection, find the primary phase current.
$\Delta-\Delta:$ primary phase current $=\frac{\mathrm{I}}{\sqrt{3}}=\frac{10.4}{1.73}=6 \mathrm{amps}$
$\mathrm{Y}-\mathrm{Y}:$ primary phase current $=\mathrm{I}=10.4 \mathrm{amps}$
$\mathrm{Y}-\Delta:$ primary phase current $=\mathrm{I}=10.4 \mathrm{amps}$
$\Delta-\mathrm{Y}:$ primary phase current $=\frac{\mathrm{I}}{\sqrt{3}}=\frac{10.4}{1.73}=6 \mathrm{amps}$
Example 3: Find the secondary line current and phase current for each type of transformer connection, if primary line current is 20 amps , and the turns ratio is $4: 1$.
$\Delta-\Delta: \quad$ secondary line current $=4(20)=80 \mathrm{amps}$
secondary phase current $=\frac{\mathrm{aI}}{\sqrt{3}}=\frac{4(20)}{1.73}=46.2 \mathrm{amps}$
Y-Y: $\quad$ second line current $=\mathrm{aI}=4(20)=80 \mathrm{amps}$
second phase current $=\mathrm{aI}=4(20)=80 \mathrm{amps}$
Y- $\Delta: \quad$ secondary line current $=\sqrt{3} \mathrm{aI}=(1.73)(4)(20)=138.4 \mathrm{amps}$
secondary phase current $=\mathrm{aI}=4(20)=80 \mathrm{amps}$
$\Delta-\mathrm{Y}: \quad$ secondary line current $=\frac{\mathrm{aI}}{\sqrt{3}}=\frac{4(20)}{1.73}=46.2 \mathrm{amps}$
secondary phase current $=\frac{\mathrm{aI}}{\sqrt{3}}=\frac{4(20)}{1.73}=46.2 \mathrm{amps}$

## Transformer Losses and Efficiency

All transformers have copper and core losses. Copper loss is power lost in the primary and secondary windings of a transformer due to the ohmic resistance of the windings. Copper loss, in watts, can be found using Equation (13-7).

$$
\begin{equation*}
\text { Copper Loss }=I_{P}^{2} R_{P}+I_{S}^{2} R_{S} \tag{13-7}
\end{equation*}
$$

where
$\mathrm{I}_{\mathrm{P}}=$ primary current
$\mathrm{I}_{\mathrm{S}}=$ secondary current
$\mathrm{R}_{\mathrm{P}}=$ primary winding resistance
$\mathrm{R}_{\mathrm{S}}=$ secondary winding resistance

Core losses are caused by two factors: hysteresis and eddy current losses. Hysteresis loss is that energy lost by reversing the magnetic field in the core as the magnetizing AC rises and falls and reverses direction. Eddy current loss is a result of induced currents circulating in the core.

The efficiency of a transformer can be calculated using Equations (13-8), (13-9), and (13-10).

$$
\begin{align*}
& \text { Efficiency }=\frac{\text { Power Output }}{\text { Power Input }}=\frac{\mathrm{P}_{\mathrm{S}}}{\mathrm{P}_{\mathrm{P}}} \times 100  \tag{13-8}\\
& \text { Efficiency }=\frac{\text { Power Output }}{\text { Power Output }+ \text { Copper Loss + Core Loss }} \times 100  \tag{13-9}\\
& \text { Efficiency }=\frac{\mathrm{V}_{\mathrm{S}} \mathrm{I}_{\mathrm{S}} \times \text { PF }}{\left(\mathrm{V}_{\mathrm{S}} \mathrm{I}_{\mathrm{S}} \times \mathrm{PF}\right)+\text { Copper Loss + Core Loss }} \times 100 \tag{13-10}
\end{align*}
$$

where
$\mathrm{PF}=$ power factor of the load

Example 1: A 5:1 step-down transformer has a full-load secondary current of 20 amps . A short circuit test for copper loss at full load gives a wattmeter reading of 100 W . If $\mathrm{R}_{\mathrm{P}}=0.3 \Omega$, find $\mathrm{R}_{\mathrm{S}}$ and power loss in the secondary.

Solution:

$$
\text { Copper Loss }=\mathrm{I}_{\mathrm{P}}^{2} \mathrm{R}_{\mathrm{P}}+\mathrm{I}_{\mathrm{S}}^{2} \mathrm{R}_{\mathrm{S}}=100 \mathrm{~W}
$$

To find $I_{p}$ :

$$
\begin{aligned}
& \frac{N_{P}}{N_{S}}=\frac{I_{S}}{I_{P}} \\
& I_{P}=\frac{N_{S}}{N_{P}} I_{S}=\frac{1}{5} 20=4 \mathrm{amps}
\end{aligned}
$$

To find $\mathrm{R}_{\mathrm{S}}$ :

$$
\begin{aligned}
I_{S}^{2} R_{S} & =100-I_{P}^{2} R_{P} \\
R_{S} & =\frac{100-I_{P}^{2} R_{P}}{I_{S}^{2}}=\frac{100-0.3(4)^{2}}{20^{2}}=0.24
\end{aligned}
$$

Power loss in secondary $=\mathrm{I}_{\mathrm{S}}{ }^{2} \mathrm{R}_{\mathrm{S}}=(20)^{2}(0.24)=96 \mathrm{~W}$
Example 2: An open circuit test for core losses in a 10 kVA transformer [Example (1)] gives a reading of 70 W . If the PF of the load is $90 \%$, find efficiency at full load.

## Solution:

$$
\begin{array}{ll}
\text { Eff. } & =\frac{\mathrm{V}_{\mathrm{S}} \mathrm{I}_{\mathrm{S}} \times \mathrm{PF}}{\left(\mathrm{~V}_{\mathrm{S}} \mathrm{I}_{\mathrm{S}} \times \mathrm{PF}\right)+\text { Copper Loss }+ \text { Core Loss }} \times 100 \\
\mathrm{~V}_{\mathrm{S}} \mathrm{I}_{\mathrm{S}} & =\text { transformer rating }=10 \mathrm{kVA}=10,000 \mathrm{VA} \\
\mathrm{PF} & =0.90 ; \text { Copper loss }=100 \mathrm{~W} ; \text { Core loss }=70 \mathrm{~W} \\
\text { Eff } & =\frac{10,000(0.90)}{10,000(0.90)+100+70} \times 100=\frac{9000}{9170} \times 100=98.2 \%
\end{array}
$$

## Transformer Operation Under No-Load

If the secondary of a transformer is left open-circuited (Figure 6), primary current is very low and is called the no-load current. No-load current produces the magnetic flux and supplies the hysteresis and eddy current losses in the core. The no-load current ( $\mathrm{I}_{\mathrm{E}}$ ) consists of two components: the magnetizing current $\left(\mathrm{I}_{\mathrm{m}}\right)$ and the core loss $\left(\mathrm{I}_{\mathrm{H}}\right)$. Magnetizing current lags applied voltage by $90^{\circ}$, while core loss is in phase with the applied voltage (Figure 6b). $\mathrm{V}_{\mathrm{P}}$ and $V_{S}$ are shown $180^{\circ}$ out of phase. $I_{H}$ is very small in comparison with $I_{m}$, and $I_{m}$ is nearly equal to $\mathrm{I}_{\mathrm{E}}$. No-load current, $\mathrm{I}_{\mathrm{E}}$, is also referred to as exciting current.


Figure 6 Open-Circuit Secondary

Example: When the secondary of a $120 / 440 \mathrm{~V}$ transformer is open, primary current is 0.2 amps at a PF of .3. The transformer is a 5 kVA transformer. Find: (a) $I_{P}$, (b) $I_{E}$, (c) $I_{H}$, and (d) $I_{m}$.
(a) Full load current $=\frac{\mathrm{kVA} \text { Rating }}{\mathrm{V}_{\mathrm{P}}}$
(b) $I_{P}$ at no load is equal to $I_{E}$

$$
\mathrm{I}_{\mathrm{E}}=0.2 \mathrm{amp}
$$

(c) $I_{H}=I_{E} \cos \theta=I_{E} \times P F$

$$
=0.2(0.3)
$$

$$
\mathrm{I}_{\mathrm{H}}=0.06 \mathrm{amps}
$$

(d) $I_{M}=I_{E} \sin \theta$

$$
\begin{aligned}
\theta & =\arccos 0.3=72.5^{\circ} \\
& =(0.2) \sin 72.5^{\circ}=(0.2)(0.95) \\
\mathrm{I}_{\mathrm{M}} & =0.19 \mathrm{amps}
\end{aligned}
$$

## Coil Polarity

The symbol for a transformer gives no indication of the phase of the voltage across the secondary. The phase of that voltage depends on the direction of the windings around the core. In order to solve this problem, polarity dots are used to show the phase of primary and secondary signals. The voltages are either in phase (Figure 7a) or $180^{\circ}$ out of phase with respect to primary voltage (Figure 7b).


Figure 7 Polarity of Transformer Coils

## Summary

The important information covered in this chapter is summarized below.

## Transformer Theory Summary

- The induction of an EMF in a coil by magnetic flux lines generated in another coil is called mutual induction.
- The turns ratio is defined as the ratio of turns of wire in the primary winding to the number of turns of wire in the secondary winding.
- The ratio between the primary and secondary impedances is referred to as the impedance ratio.
- Efficiency of a transformer is the ratio of the power output to the power input.
- In a delta connection, all three phases are connected in series to form a closed loop.
- In a wye connection, three common ends of each phase are connected together at a common terminal, and the other three ends are connected to a three-phase line.
- In a $\Delta$ connected transformer:

$$
\begin{aligned}
\mathrm{V}_{\mathrm{L}} & =\mathrm{V} \phi \\
\mathrm{I}_{\mathrm{L}} & =\sqrt{3} \mathrm{I} \phi
\end{aligned}
$$

- In a Y connected transformer:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{L}}=\sqrt{3} \mathrm{~V} \phi \\
& \mathrm{I}_{\mathrm{L}}=\mathrm{I} \phi
\end{aligned}
$$

## TRANSFORMER TYPES

Transformers can be constructed so that they are designed to perform a specific function. A basic understanding of the various types of transformers is necessary to understand the role transformers play in today's nuclear facilities.

EO 1.4 STATE the applications of each of the following types of transformers:
a. Distribution
b. Power
c. Control
d. Auto
e. Isolation
f. Instrument potential
g. Instrument current

## Types of Transformers

Transformers are constructed so that their characteristics match the application for which they are intended. The differences in construction may involve the size of the windings or the relationship between the primary and secondary windings. Transformer types are also designated by the function the transformer serves in a circuit, such as an isolation transformer.

## Distribution Transformer

Distribution transformers are generally used in electrical power distribution and transmission systems. This class of transformer has the highest power, or volt-ampere ratings, and the highest continuous voltage rating. The power rating is normally determined by the type of cooling methods the transformer may use. Some commonly-used methods of cooling are by using oil or some other heat-conducting material. Ampere rating is increased in a distribution transformer by increasing the size of the primary and secondary windings; voltage ratings are increased by increasing the voltage rating of the insulation used in making the transformer.

## Power Transformer

Power transformers are used in electronic circuits and come in many different types and applications. Electronics or power transformers are sometimes considered to be those with ratings of 300 volt-amperes and below. These transformers normally provide power to the power supply of an electronic device, such as in power amplifiers in audio receivers.

## Control Transformer

Control transformers are generally used in electronic circuits that require constant voltage or constant current with a low power or volt-amp rating. Various filtering devices, such as capacitors, are used to minimize the variations in the output. This results in a more constant voltage or current.

## Auto Transformer

The auto transformer is generally used in low power applications where a variable voltage is required. The auto transformer is a special type of power transformer. It consists of only one winding. By tapping or connecting at certain points along the winding, different voltages can be obtained (Figure 8).


Figure 8 Auto Transformer Schematic

## Isolation Transformer

Isolation transformers are normally low power transformers used to isolate noise from or to ground electronic circuits. Since a transformer cannot pass DC voltage from primary to secondary, any DC voltage (such as noise) cannot be passed, and the transformer acts to isolate this noise.

## Instrument Potential Transformer

The instrument potential transformer (PT) steps down voltage of a circuit to a low value that can be effectively and safely used for operation of instruments such as ammeters, voltmeters, watt meters, and relays used for various protective purposes.

## Instrument Current Transformer

The instrument current transformer (CT) steps down the current of a circuit to a lower value and is used in the same types of equipment as a potential transformer. This is done by constructing the secondary coil consisting of many turns of wire, around the primary coil, which contains only a few turns of wire. In this manner, measurements of high values of current can be obtained.

A current transformer should always be short-circuited when not connected to an external load. Because the magnetic circuit of a current transformer is designed for low magnetizing current when under load, this large increase in magnetizing current will build up a large flux in the magnetic circuit and cause the transformer to act as a step-up transformer, inducing an excessively high voltage in the secondary when under no load.

## Summary

The important information covered in this chapter is summarized below.

## Transformer Types Summary

- Distribution transformers are generally used in power distribution and transmission systems.
- Power transformers are used in electronic circuits and come in many different types and applications.
- Control transformers are generally used in circuits that require constant voltage or constant current with a low power or volt-amp rating.
- Auto transformers are generally used in low power applications where a variable voltage is required.
- Isolation transformers are normally low power transformers used to isolate noise from or to ground electronic circuits.
- Instrument potential and instrument current transformers are used for operation of instruments such as ammeters, voltmeters, watt meters, and relays used for various protective purposes.

